

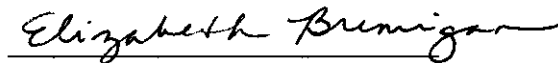
A COMPILATION OF GEOMETRY TEACHING RESOURCES

An Honors Thesis (HONRS 499)

By

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Thesis Advisor
Dr. Elizabeth Bremigan

A handwritten signature in cursive script, reading "Elizabeth Bremigan", is positioned above a horizontal line.

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Muncie, Indiana

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ABSTRACT

Finding and gathering resources to use in a classroom is time consuming and it is often difficult for beginning teachers to find the time to bring in outside resources. In addition, veteran teachers are constantly looking for new materials and resources to incorporate into the classroom. This collection of geometry resources has been compiled with these two audiences in mind. New teachers will find extra activities that can easily supplement the textbook. Veteran teachers may find some interesting activities that they may not have seen before.

The activities contained in this project are broken into eight categories: visualization/planning/reasoning/critical thinking, lines, triangles, polygons, circles, solids, projects and miscellaneous resources. Each activity is in a format that can be ready for a classroom. In addition, if appropriate, each activity has an answer sheet following immediately. Included in this project is a CD with all of the activities so that a teacher using these resources can easily edit an activity to fit their individual classroom.

ACKNOWLEDGEMENTS

- I would like to thank my advisor, Dr. Elizabeth Bremigan, for being willing to accept my idea and help me form it into a workable project.
- I would also like to extend my appreciation to Dr. David Thomas for his inspiration in my honors thesis.
- I would like to thank Hannah Bouslough for her support and encouragement during the formulation and creation of my thesis.
- In addition, I would like to thank classroom teachers that I consulted throughout my thesis work: Heather Hudson of Jay County High School, Julie Carlin of Jay County High School and Vicky Hamen of Celina High School.

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- Bibliography
- A CD of the word documents containing the activities is in the front pocket of the binder.

***Each section contains its own bibliography to help pinpoint where the activity was originally found.**

Dear Reader,

During the years that I have studied mathematics and mathematics education one aspect that I have noticed is some, not all, students and future teachers alike seem to dread the subject of geometry. To me, it seems geometry has gotten a bad reputation. I decided to collect geometry resources for teachers because this is one area I felt that I have always lacked mathematical knowledge and feel this project will help expand my knowledge and help other teachers at the same time. In addition, there are so many resources for teaching geometry that it is hard to decipher what is helpful in a classroom and to a student.

One of my guiding thoughts during my thesis work is a theory based on the research conducted by Dina and Pierre van Heile. During the 1950's these Dutch educators developed a model of the levels a student will naturally progress through during their geometric education. The van Heile Model is based on five levels of geometric understanding. At the *Basic* or *Holistic* level students recognize basic shapes but do not connect properties with shapes. In the *Analysis* level students are able to make connections between shapes and their properties. However, students are not able to make connections between shapes, for example, they do not realize that a square and a rectangle have a relationship. In the next level, *Informal Deduction*, students start to understand relationships between properties and figures and the relationships among figures. In this level students never progress beyond the ability to follow informal proofs and are unable to construct proofs from unfamiliar starting points. In the *Deductive Reasoning* level students develop the ability to follow proofs and can deduce one statement from another. The last and highest level is *Rigor*. Few people ever achieve this level of geometric understanding. At the rigor level proofs and statements can be made and justified without concrete examples. Most people never proceed past the *Informal Deduction* level, which is where most high school geometry classes fall. Using the van Heile Model as a basis, it seems unreasonable to expect high school students to be able to do more than informally prove statements. Therefore, I have focused my thesis work on easy to follow activities that do not require students to be able to formally prove a statement.

I hope that in my research I have uncovered and provided some helpful and fun activities for students and teachers alike. This collection of resources is far from being complete, but hopefully brings together some informative and imaginative activities to use in a classroom. Although geometry is a difficult subject and often requires students to think in a different manner, it is still an important subject to be studied. The study of geometry can help students develop logical thinking processes.

BRIEF HISTORY OF GEOMETRY

Evidence of geometric thinking can be dated as far back as the Egyptians. It is believed they used geometric ideas to survey land and during construction projects. In addition, evidence shows the Babylonians had some knowledge of geometric relationships, namely the three, four, five Pythagorean relationship. The famous Euclid wrote his book *The Elements*, founding rules of logic, during the time of the Greeks. This created the most well known and used geometry, Euclidean geometry. Euclidean geometry is based on five postulates. The most controversial of the five is the last postulate known as the parallel postulate.

Two main rules are used in geometry. There are basic assumptions that seem obvious and therefore require no proof. These are known as postulates or axioms. The second type of rule is known as a theorem. These rules are not necessarily as obvious and therefore require proving before use.

Besides Euclidean geometry, there are a few other forms of geometric thinking. I will not discuss these extensively because most high school classes will not even mention their existence. Non-Euclidean geometries are based only on Euclid's first four postulates. Hyperbolic and spherical geometry explore a geometric world where there are no parallel lines. There are two more possible geometric worlds to consider. Differential geometry uses calculus techniques to explain geometric happenings on a curved surface. The most recent geometrical world to develop is fractal geometry. Fractal geometry uses the aid of computer calculations to explore figures that model natural structures that have some repeating patterns, like fern leaves.

VISUALIZATION/PLANNING/REASONING/CRITICAL THINKING

- Making Predictions
 - be able to visualize what is happening and what will happen
 - help students predict what needs to happen and how to make something happen
- The Gardener
 - has students use the fact that a line is defined by three or more points
 - investigates the formation of lines
- Make Your Own Dice
 - developing spatial sense
- Same Cube?
 - developing spatial sense without a physical model to manipulate
- Hex
 - critical thinking and planning
- Dancing Knights
 - critical thinking and planning
- Architectural Plans
 - develop spatial sense with manipulative
- Tri-Bar
 - practice multiple perspectives
- Bulletin Board
 - exposes students to “strange” pictures that makes them question the reality of them
 - develop the ability to question pictures and drawings

VISUALIZATION/PLANNING/REASONING/CRITICAL THINKING BIBLIOGRAPHY

Bolt, Brian. *Mathematical Activities: A resource book for teachers*. New York, NY: Press Syndicate of the University of Cambridge, 1982.

Bruno's Tri-Bar. 12 July 2002 <http://www.SandlotScience.com>.

Ernst, Bruno. *The Magic Mirror of M. C. Escher*. Singapore: Taschen America L.L.C., 1994.

Jennings, Terry. *101 Amazing Optical Illusions: Fantastic Visual Tricks*. New York: Sterling Publishing Co., Inc., 1998.

Sturgis, Alexander. *Optical Illusions in Art*. New York: Sterling Publishing Co., Inc., 1996.

Wick, Walter. *Optical Tricks*. New York: Scholastic Inc, 1998.

MAKING PREDICTIONS

Determine how to place twelve knights on a chessboard so that every square is either occupied or attacked. (A knight moves in an “L” formation. Either two places over and one up or down, or one up or down and two left or right.)

Example of how a knight can move.

	Start			
	1			
	2	End		

[illegible]

MAKING PREDICTIONS - ANSWERS

Determine how to place twelve knights on a chessboard so that every square is either occupied or attacked. (A knight moves in an “L” formation. Either two places over and one up or down, or one up or down and two left or right.)

Example of how a knight can move.

	Start			
	1			
	2	End		

					HERE		
	HERE	HERE		HERE	HERE		
		HERE					
					HERE		
		HERE	HERE		HERE	HERE	
		HERE					

THE GARDENER

A gardener wants to make the most of the plants he has and one day he found, when laying out a rose bed, that he had managed to plant seven rose bushes in such a way that they formed six lines with three rose bushes in each line. How did he do it?

Pleased with himself the gardener looked for other interesting arrangements until he found a way of planting ten rose bushes so that he had five lines with four rose bushes in each line.

Find his two arrangements.

Now see if you can find another arrangement for his garden.

THE GARDENER - ANSWER

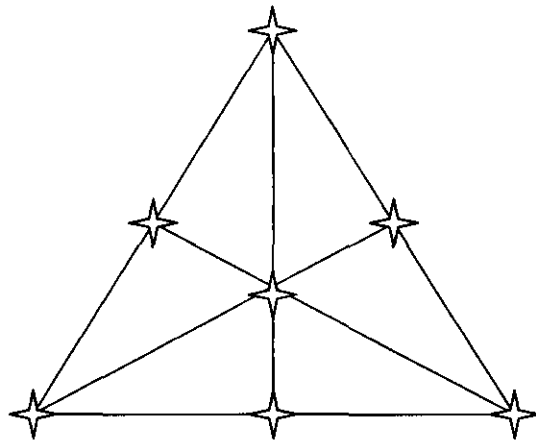
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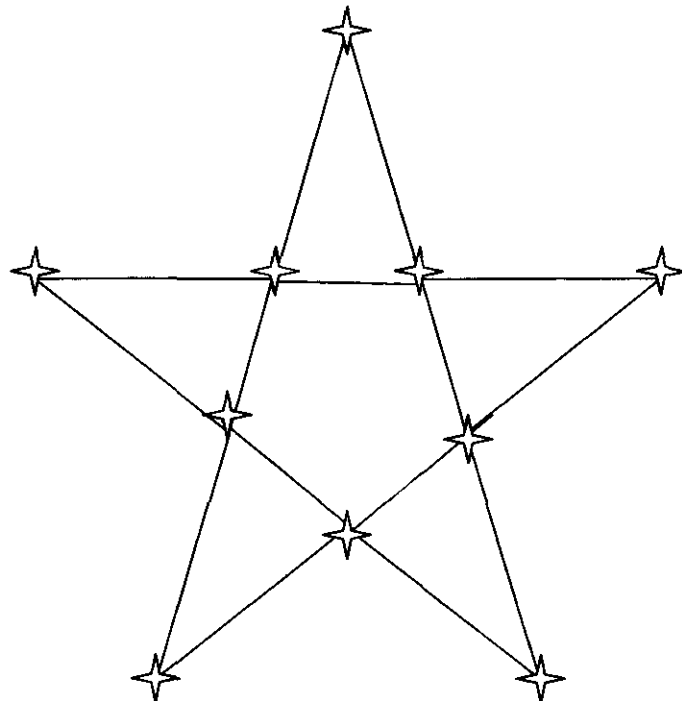
Find his two arrangements.

Now see if you can find another arrangement for his garden.

First Arrangement



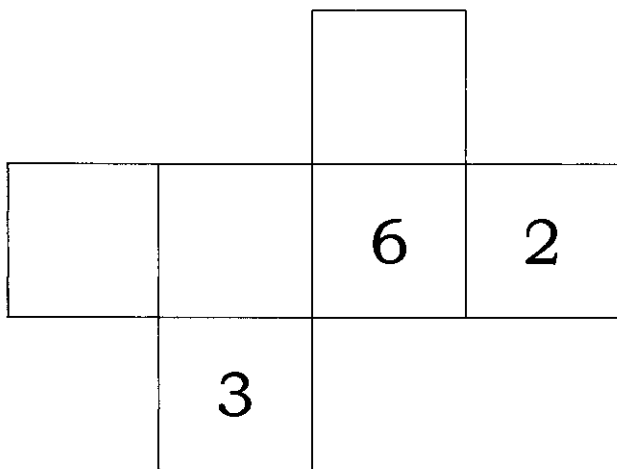
Second Arrangement



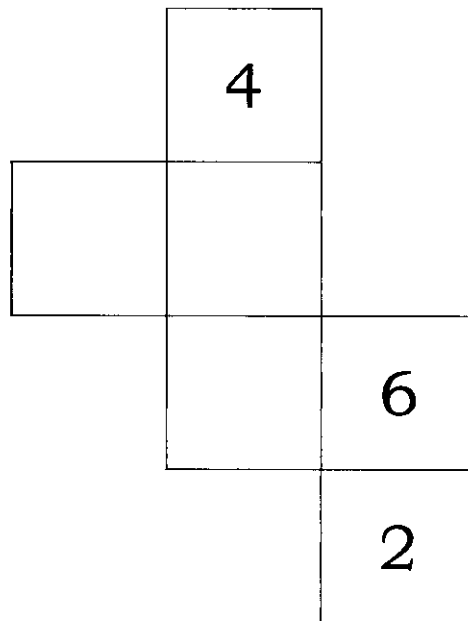
MAKE YOUR OWN DICE

Each of the three shapes shown can be folded up to make a die. In each case three of the numbers are missing. Show how to number the squares correctly so that the numbers on the opposite faces of the cube add up to 7. It might be helpful to actually cut out the shapes and create each die and then recreate the shape on a separate piece of paper.

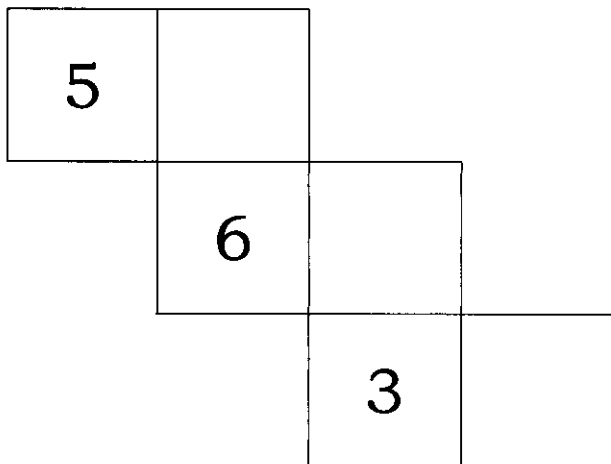
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c.

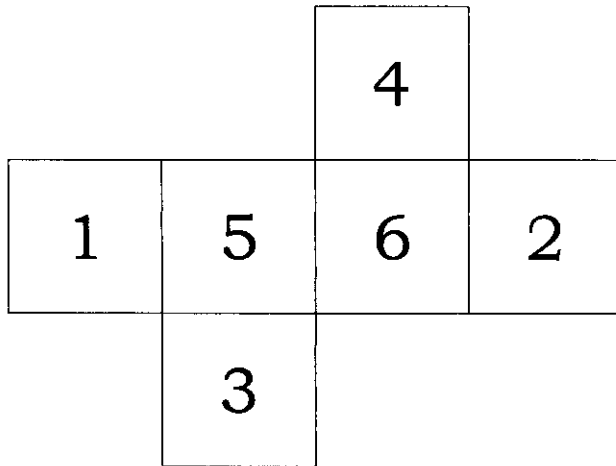


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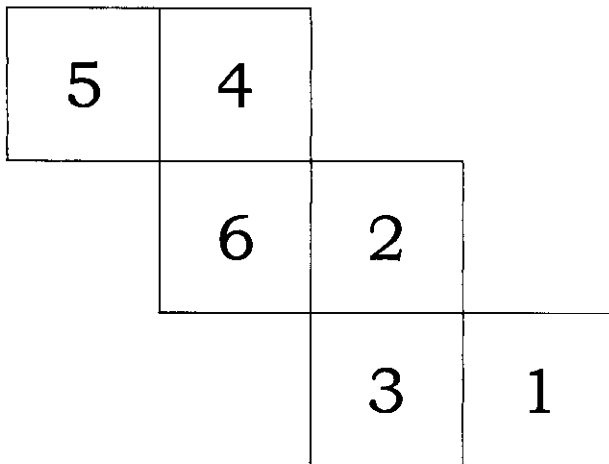


MAKE YOUR OWN DICE - ANSWERS

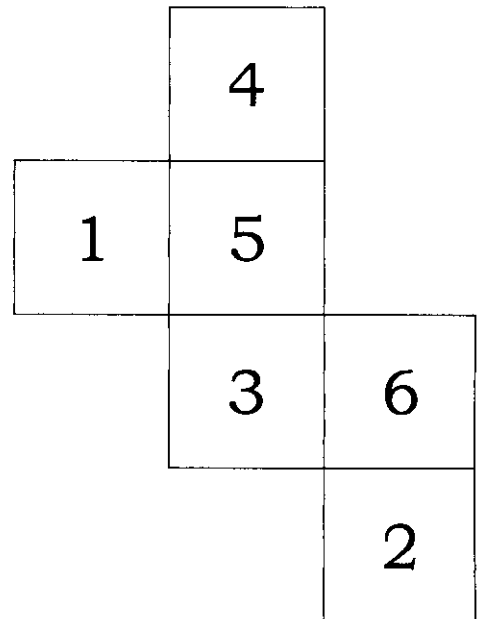
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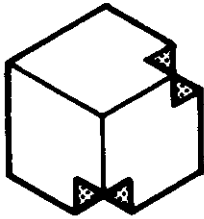


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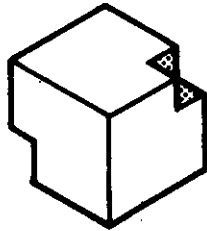


SAME CUBE?

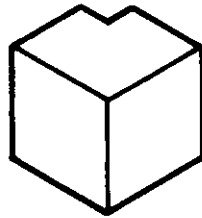
Some corners are cut out of four wooden cubes. Afterward only two of the solids formed are the same shape. Which two are they?



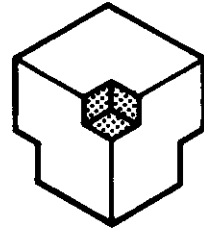
A



B



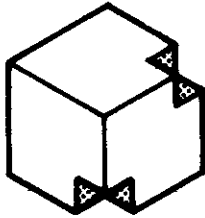
C



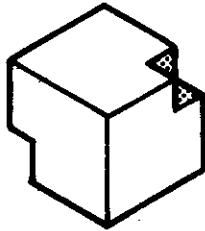
D

SAME CUBE? - ANSWER

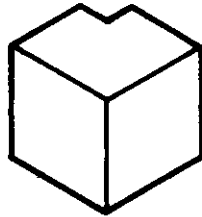
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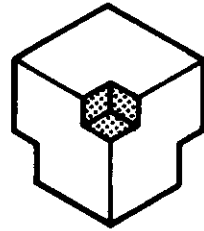
A



B



C



D

Cube A and D are the same.

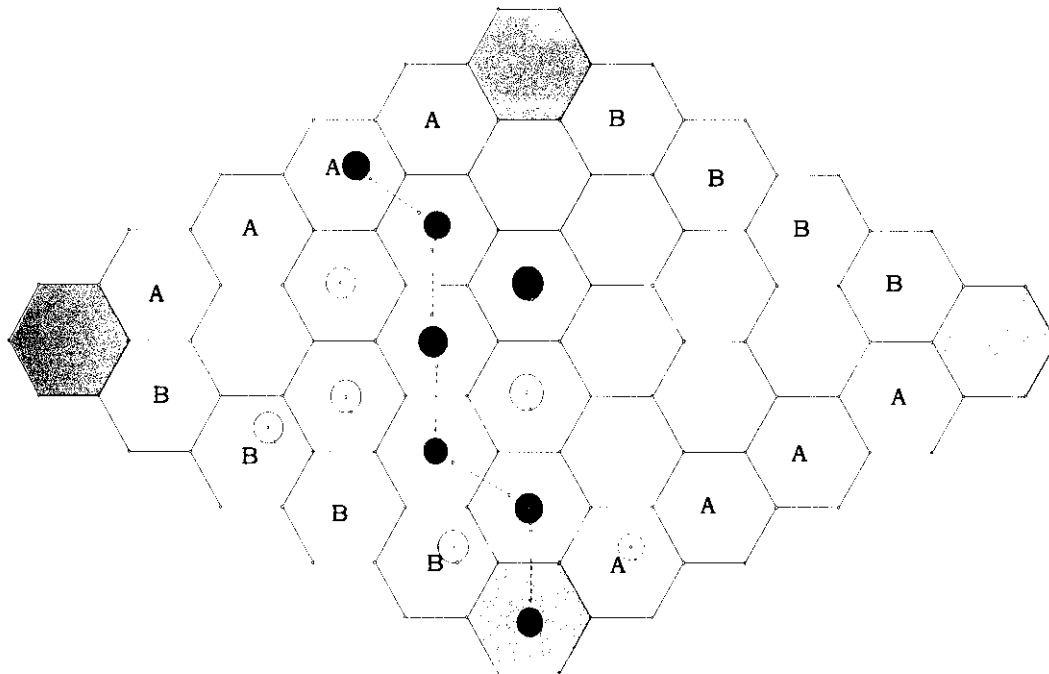
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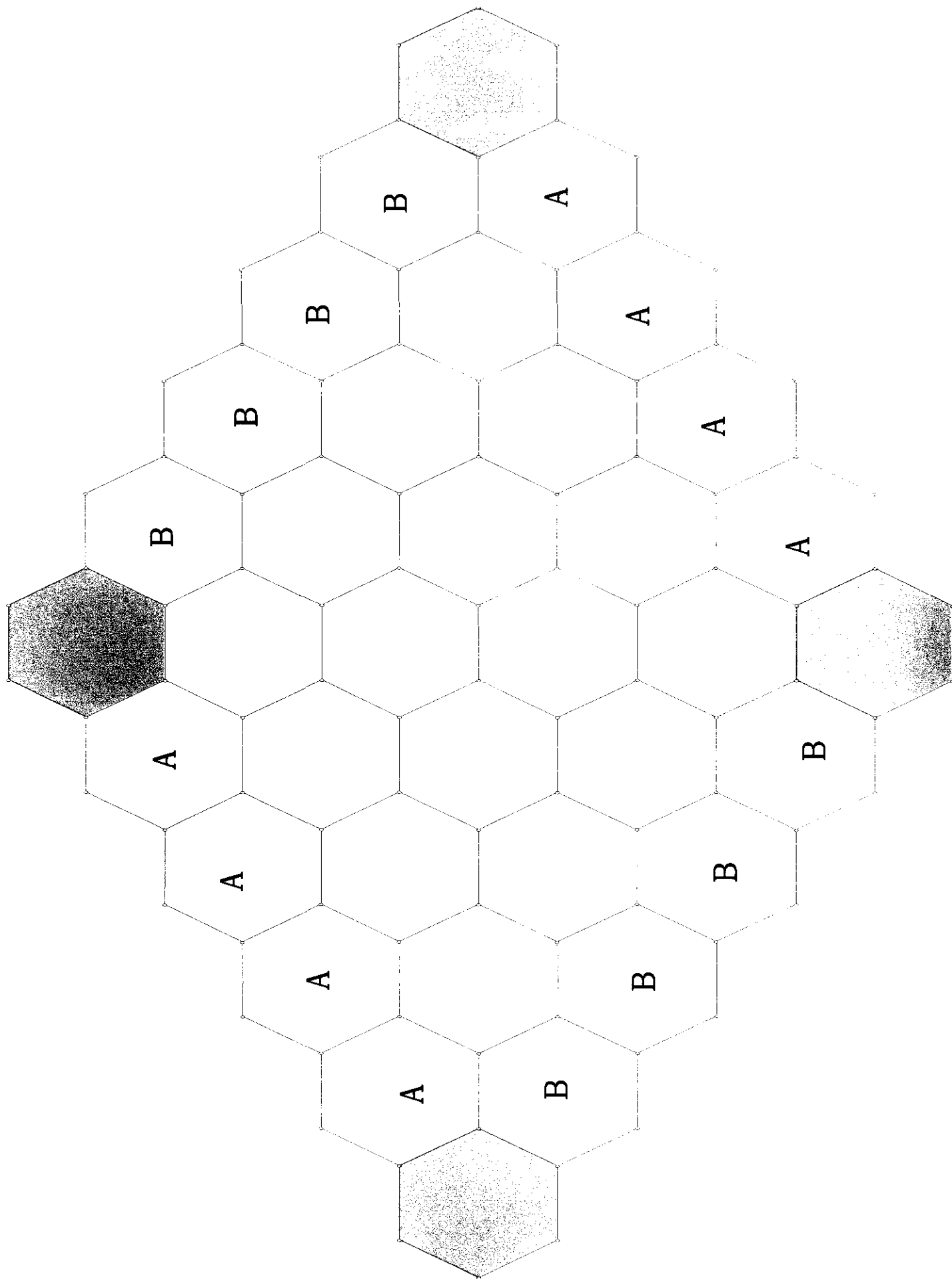
Created by the Danish mathematician Piet Hein in 1942

The game is played on a diamond shaped board (rhombus) containing interlocking hexagons. This board has six hexagons on each side but experts play on boards of up to eleven hexagons on each side.

Play: One player has a supply of black counters and the other player has a supply of white counters. Each player alternates placing a counter on an unoccupied space. A player wins when they reach the other side of the board. "Black" plays from A to A and "White" plays from B to B. The shaded hexagons can be excluded from play or can be considered either an "A" space or a "B" space.

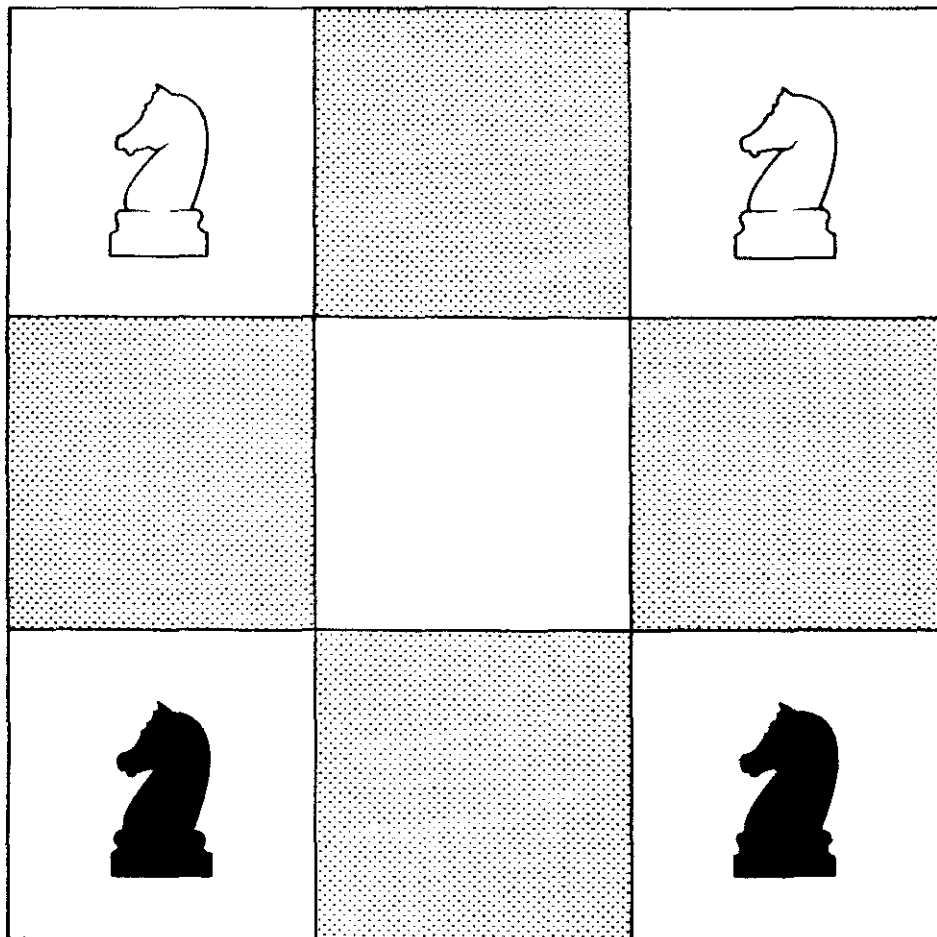
Example of "Black" winning.





DANCING KNIGHTS

Objective: Move the knights so that the black knights are where the white knights are and so that the white knights are where the black knights are in the fewest moves as possible. (A knight moves in an “L” formation, either two places over and one up or down, or one up or down and two left or right.)

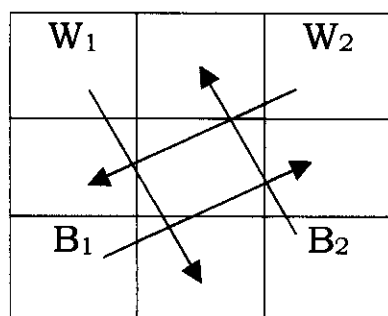


DANCING KNIGHTS - ANSWERS

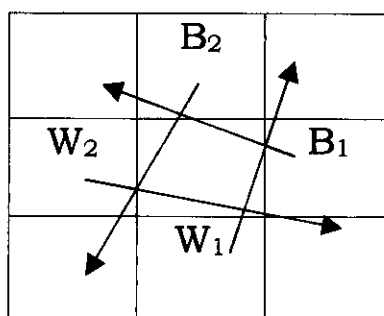
Required Moves: 16

These moves are best thought of as four groups of moves in which the four knights move from corner squares to middle squares, and vice versa in a kind of square dance in which they rotate as a foursome about the center square.

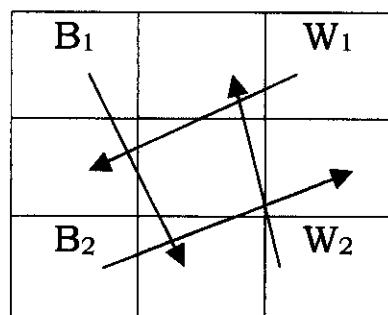
Set 1



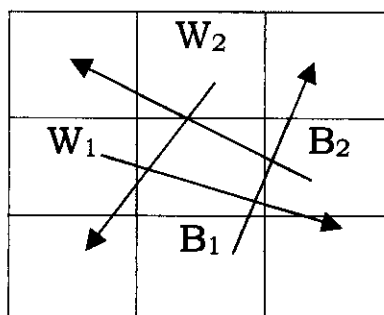
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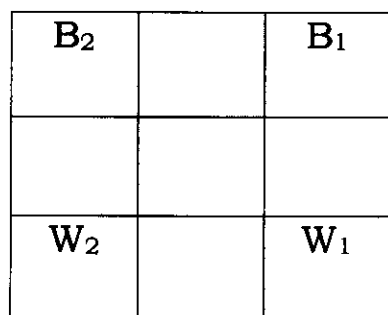
Set 3



Set 4



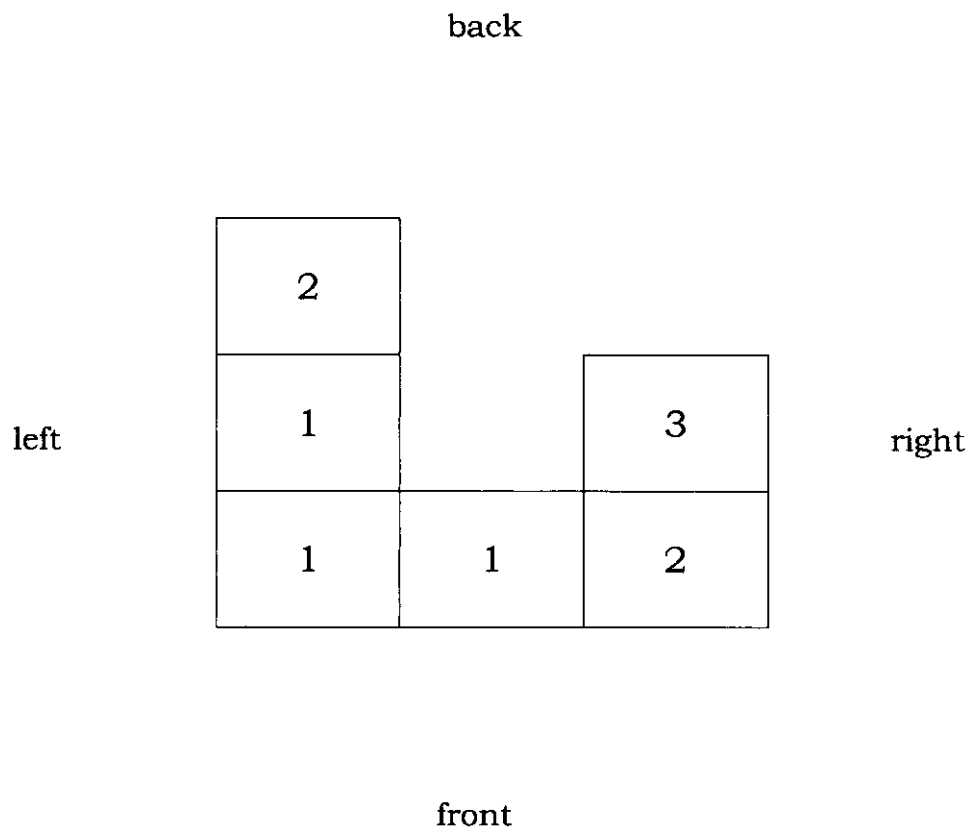
End Result



ARCHITECTURAL PLANS

Architectural plans may include various views of a building: top, front, back, left and right. By viewing a building from the top and sides, you can determine its shape. Each number on a building mat tells you the number of stories (cubes) in that section of the building.

1. Use the numbers on this mat to construct the building with your cubes.



2. Determine which views below represent the

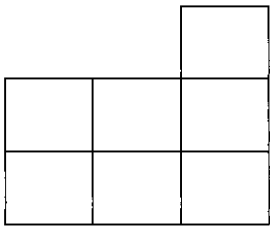
front _____

back _____

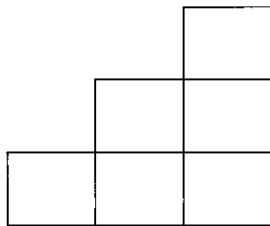
right _____

left _____

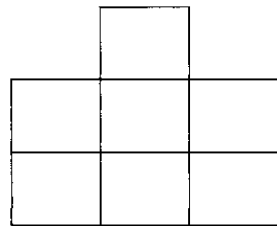
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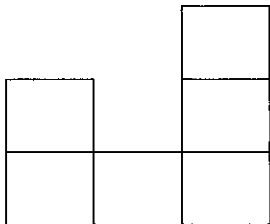
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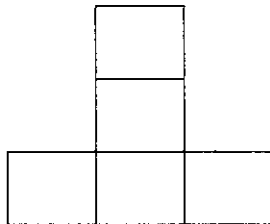
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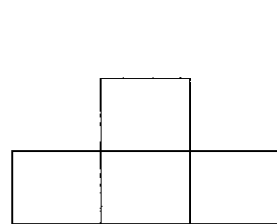
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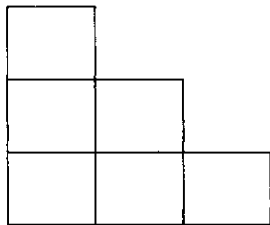
E.



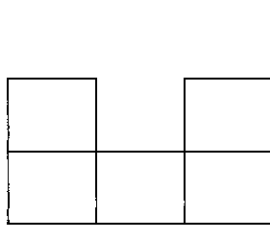
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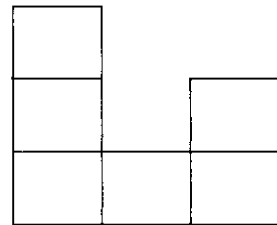
G.



H.



I.



3.

A. Use your cubes to construct the building represented by the following mats.



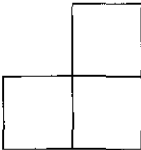
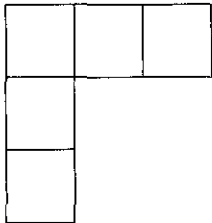
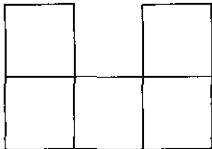
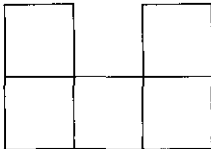
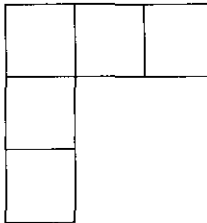
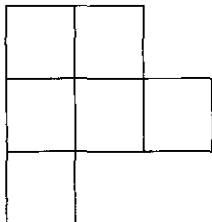
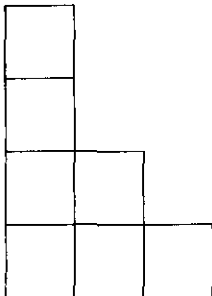
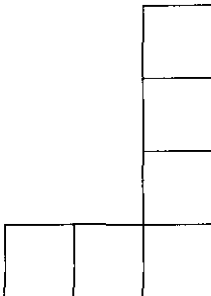
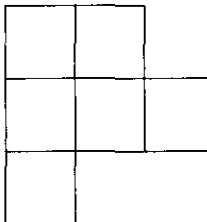
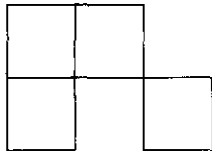
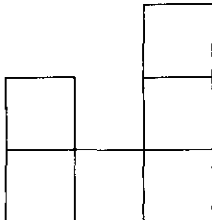
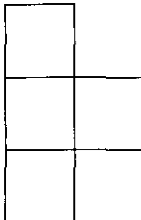
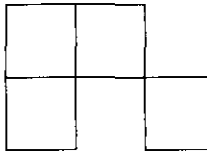
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B. On grid paper, draw the architectural plans for each building. Label the top, front, back, left and right view for each.

C. What is the relationship between the front and back views? Between the left and right views?

4. Use the plans below to construct each building. Record the height of each section of the building on the mat.

Building Views

	TOP	FRONT	RIGHT	MAT						
Example				<table border="1" style="display: inline-table; text-align: center; vertical-align: middle;"> <tr><td>2</td><td>1</td><td>1</td></tr> <tr><td>1</td><td></td><td></td></tr> </table>	2	1	1	1		
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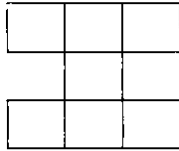
5. Going Further: Draw a set of plans for a building showing the top, front, and one side view. The building can use no more than 20 cubes. Give the plans to a classmate to construct.

ARCHITECTURAL PLANS - ANSWERS

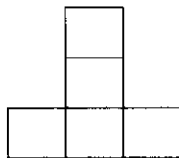
2. front: D back: I right/left: C

3.

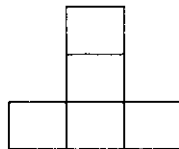
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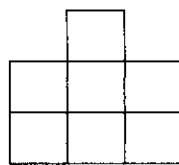
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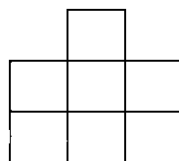
Back



Right



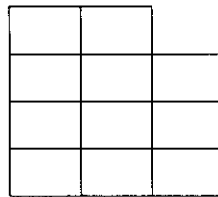
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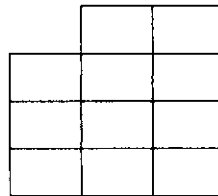
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Top



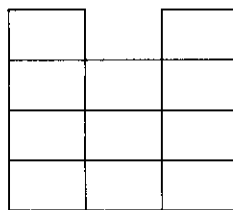
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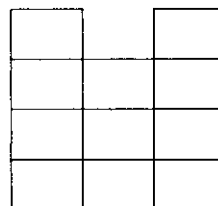
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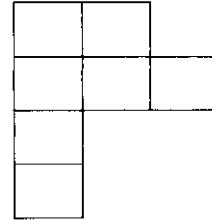
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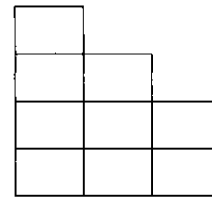
Left



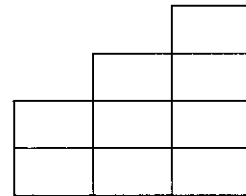
C
Top



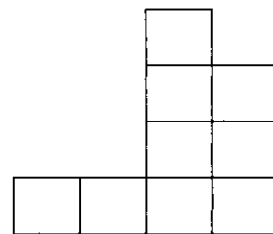
Front



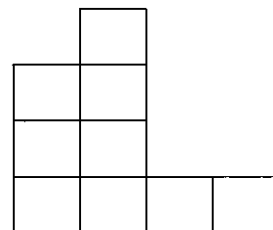
Back



Right



Left



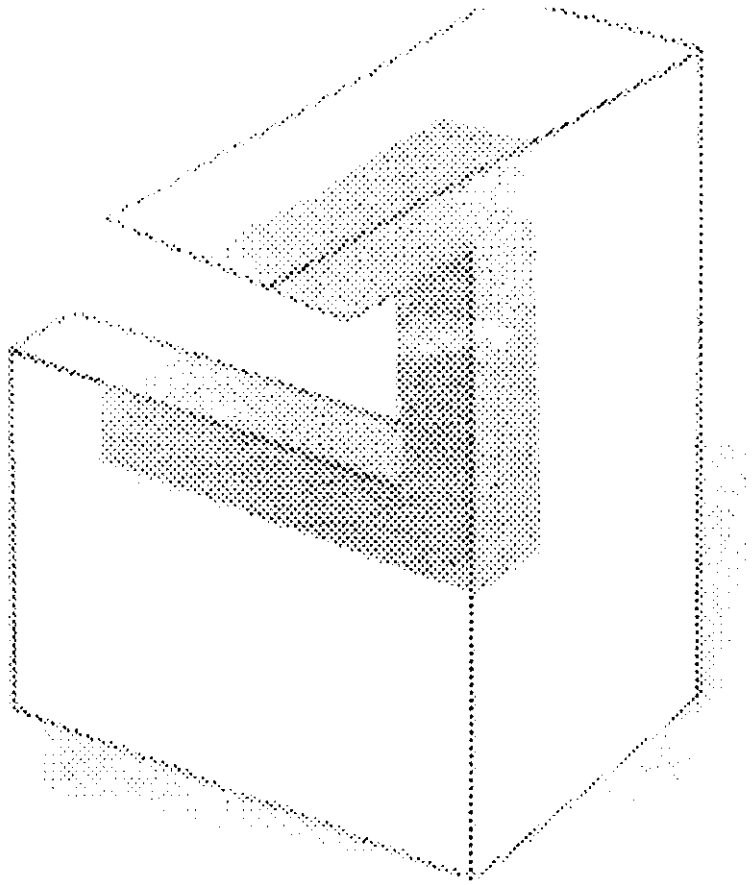
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Building Views

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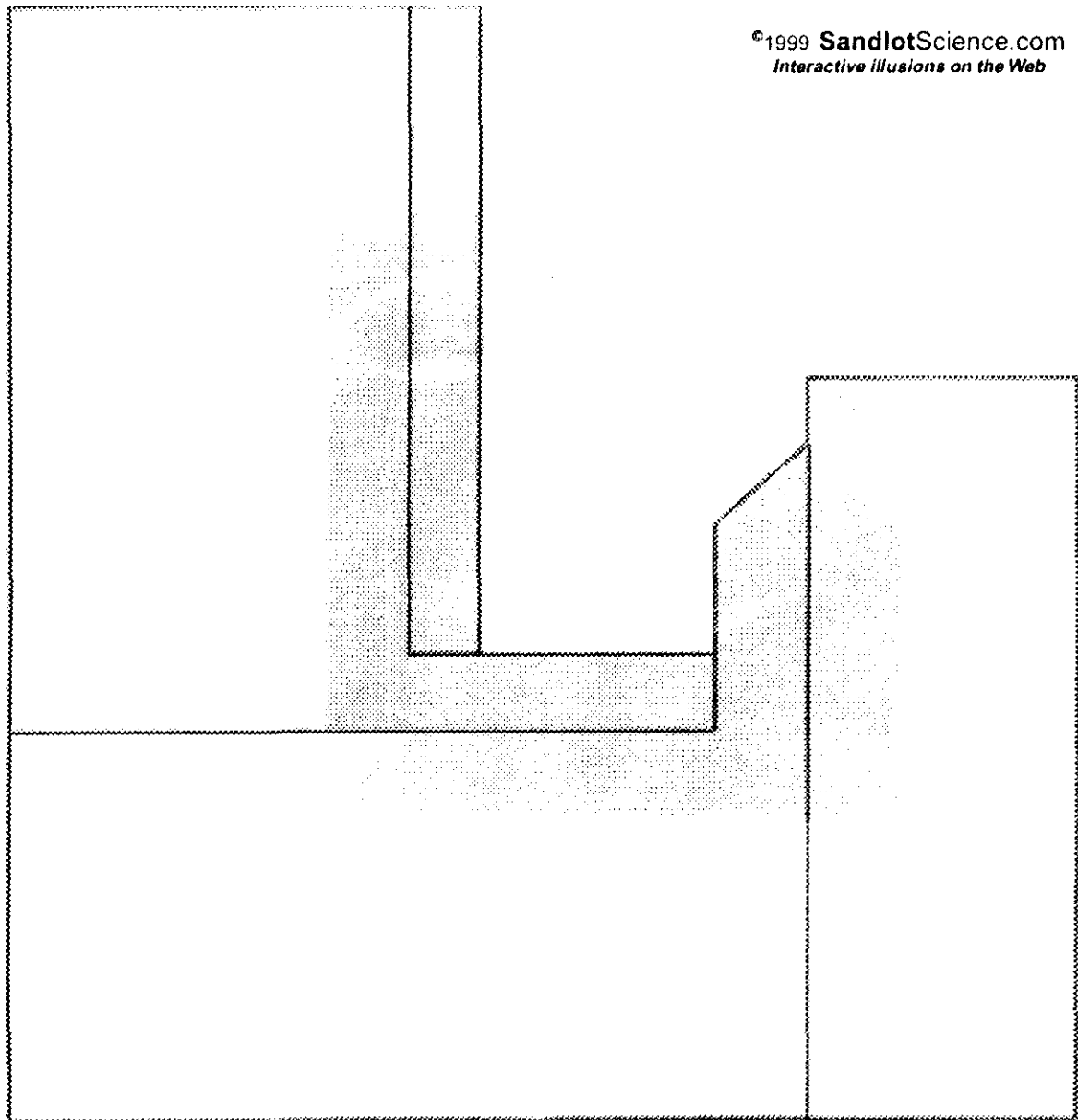
TRI-BAR

This is an “impossible” triangle. The trick is to view this triangle from the correct vantage point. Cut around the outside of the large figure below and assemble it as shown in the small picture. Adjust your view point until you “see” the triangle.



Set up your triangle as shown above.

Cut out the figure below. Notice that you must make cuts extending into the model in two places.

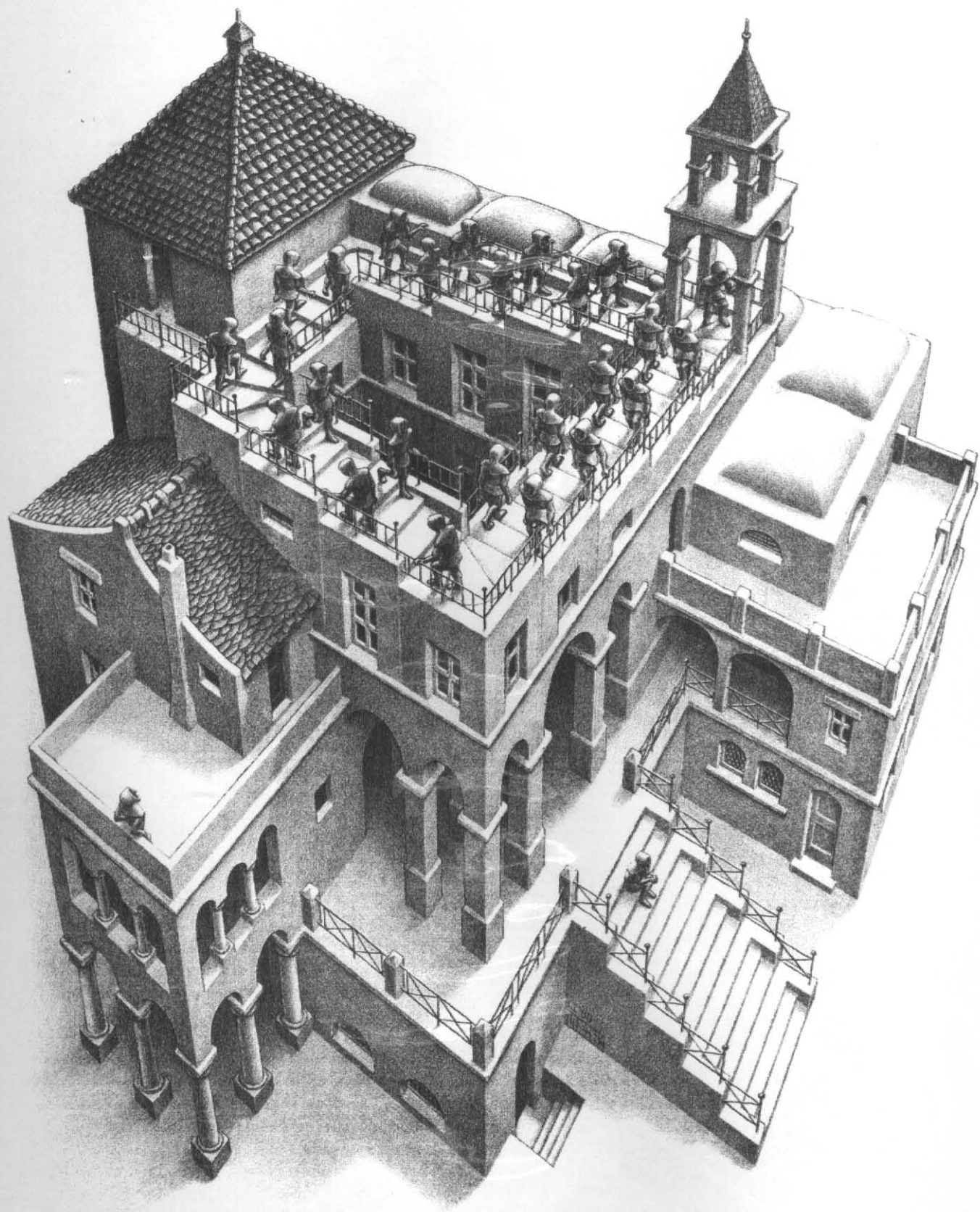


BULLETIN BOARD

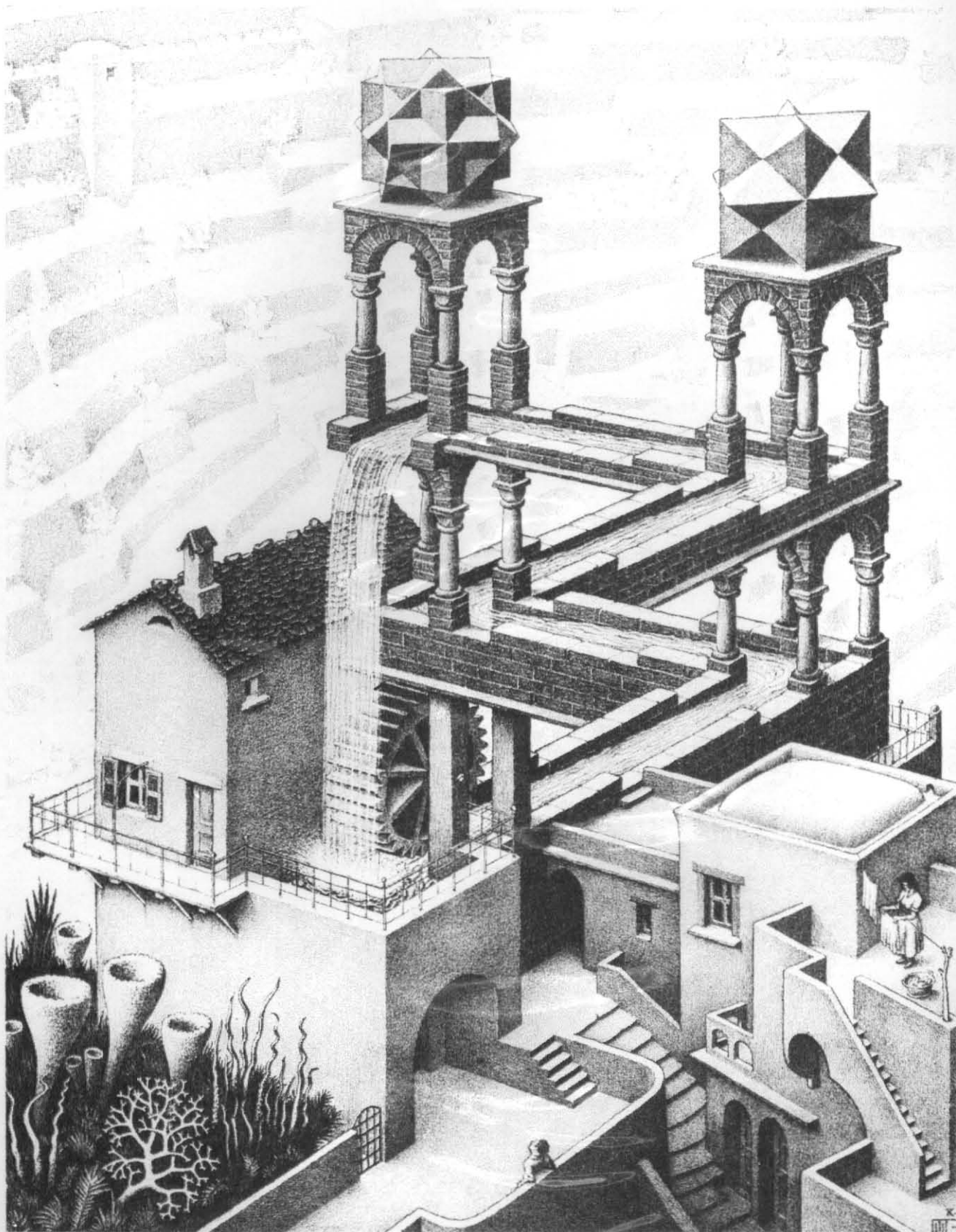
Here are pictures and brain/mind/eye puzzles to present to students. These pictures would work well as a bulletin board in your classroom to make students question the pictures and diagrams they see in their textbooks, but with a little spin.



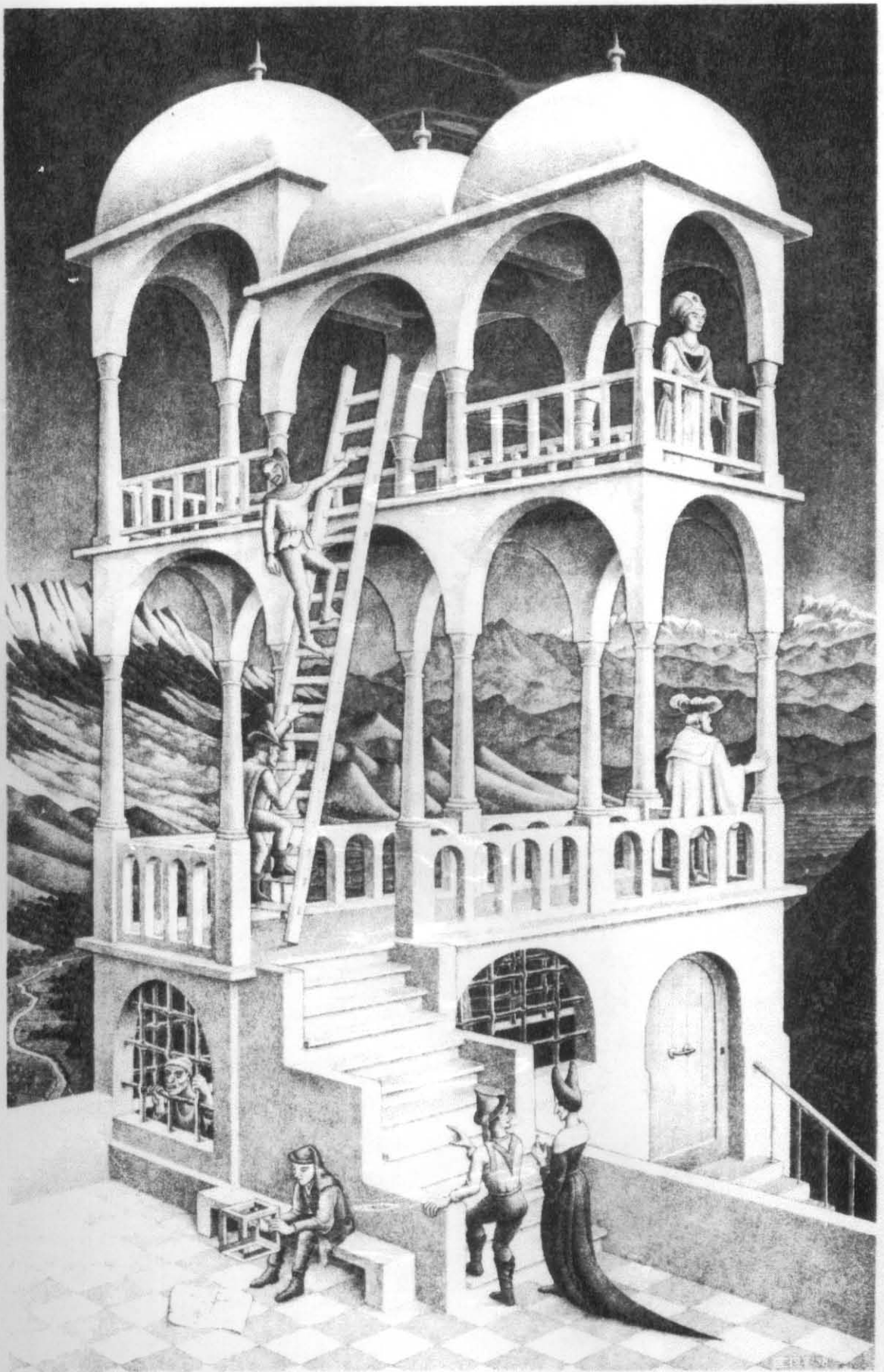
False Perspective, William Hogarth, 1754



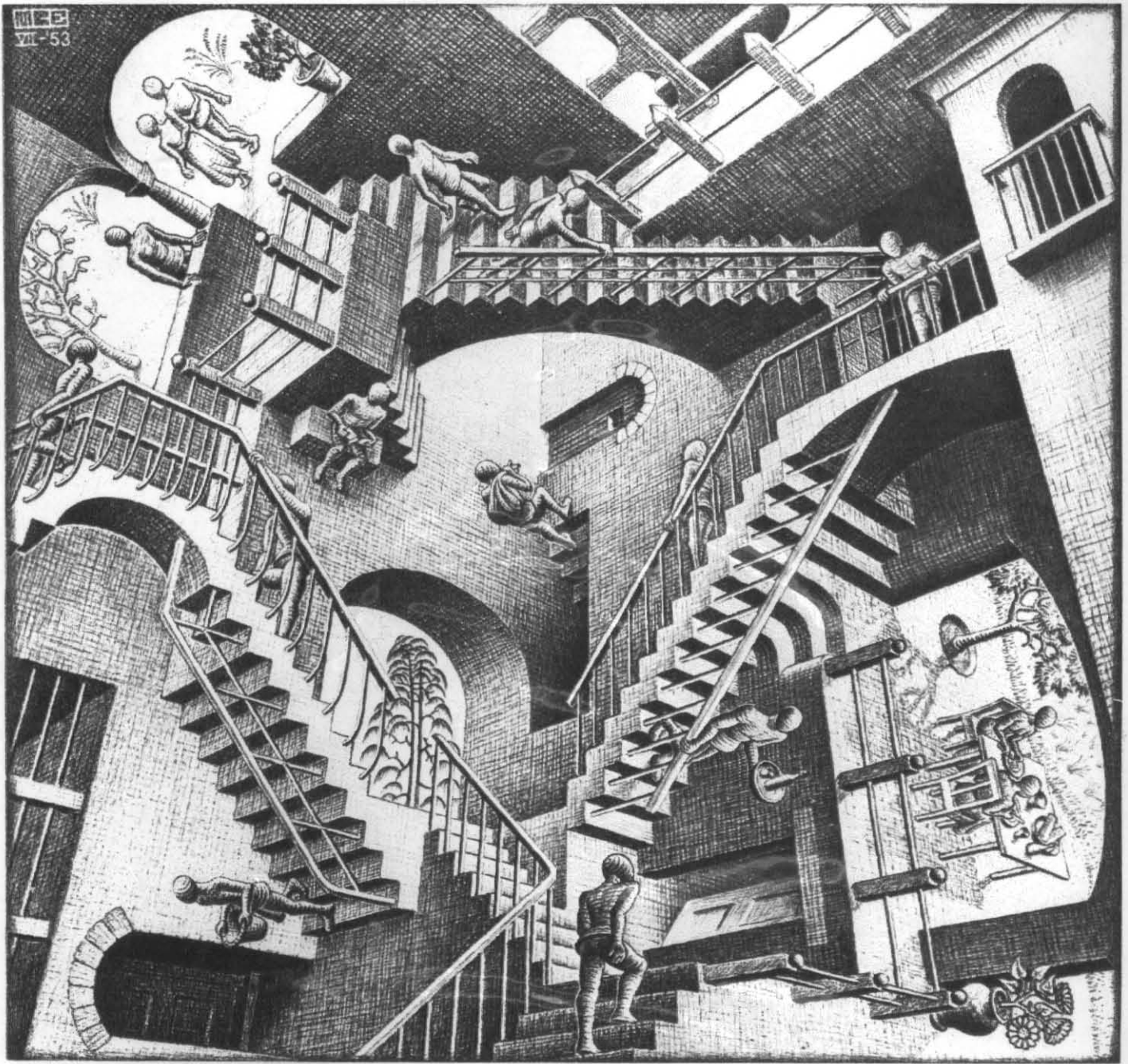
Ascending and Descending, M. C. Escher, lithograph, 1960



Waterfall, M. C. Escher, lithograph, 1961



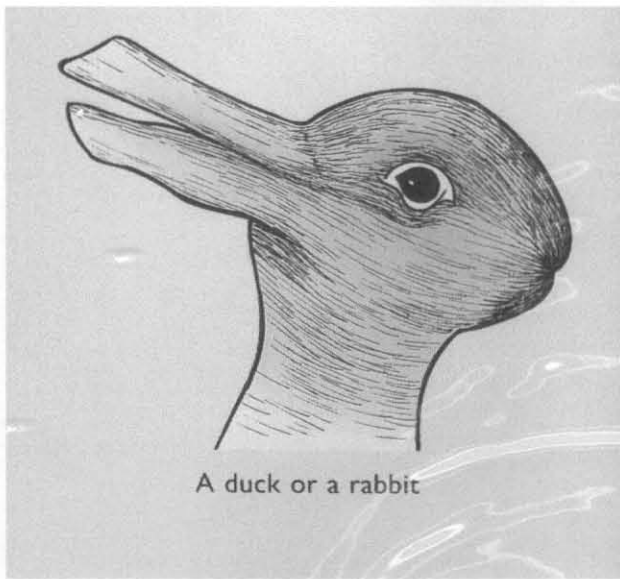
Belvedere, M. C. Escher, lithograph, 1958



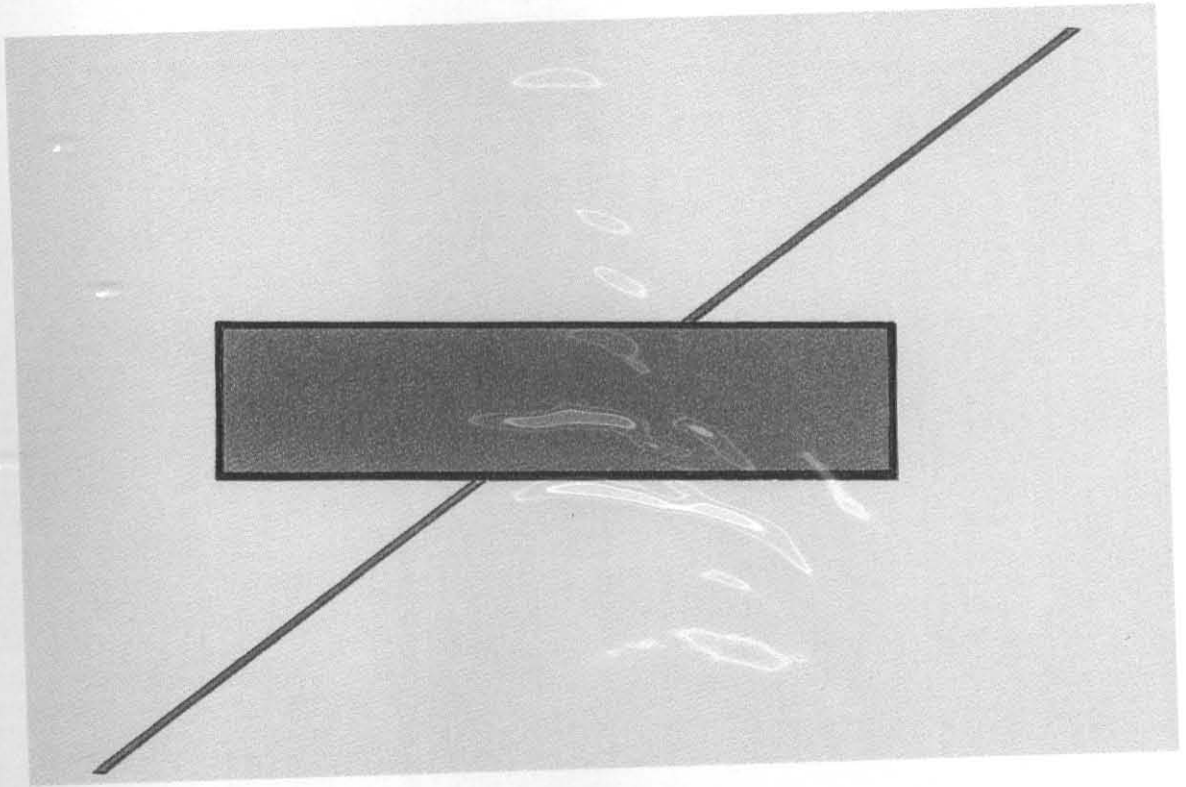
Relativity, M. C. Escher, lithograph, 1953



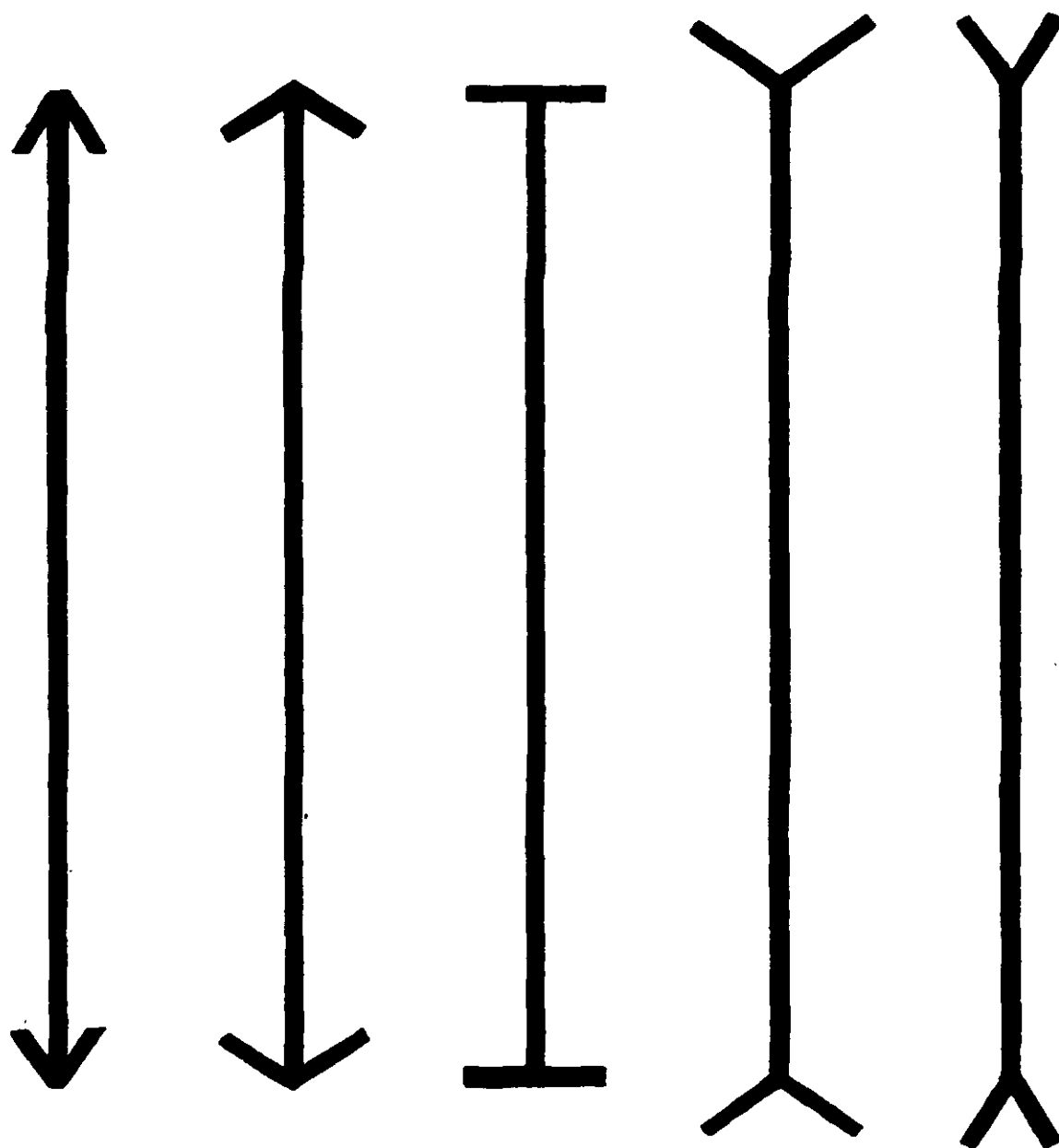
How old is the woman in this picture?



A duck or a rabbit

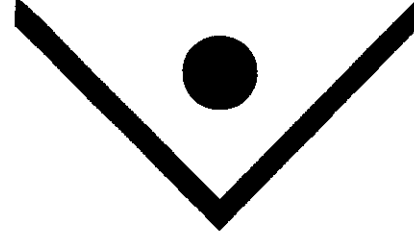
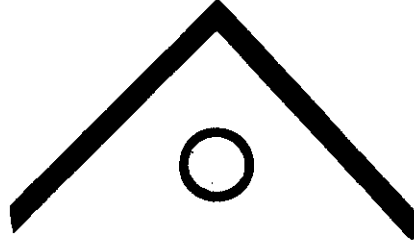
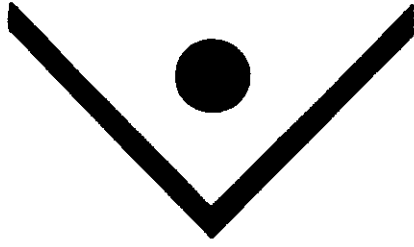


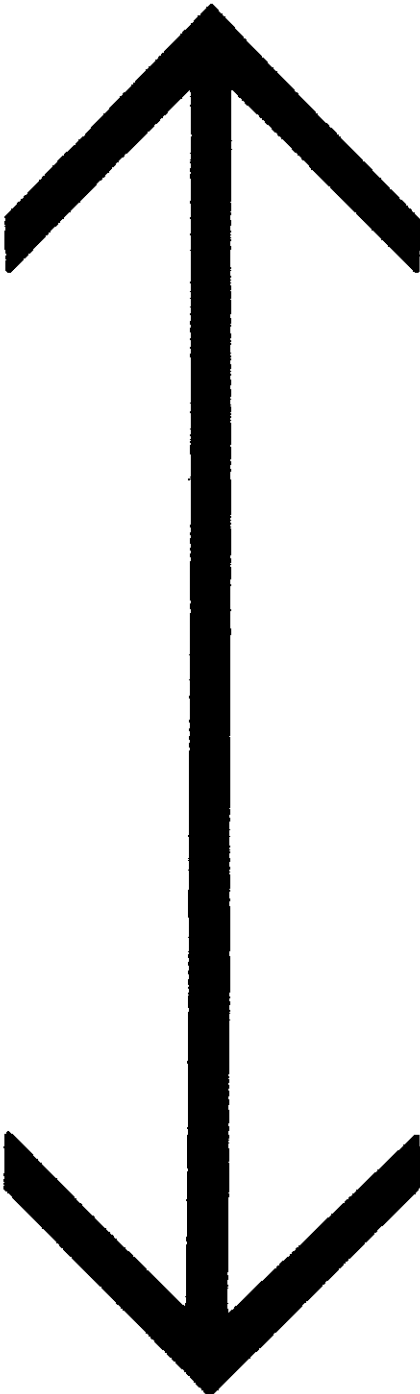
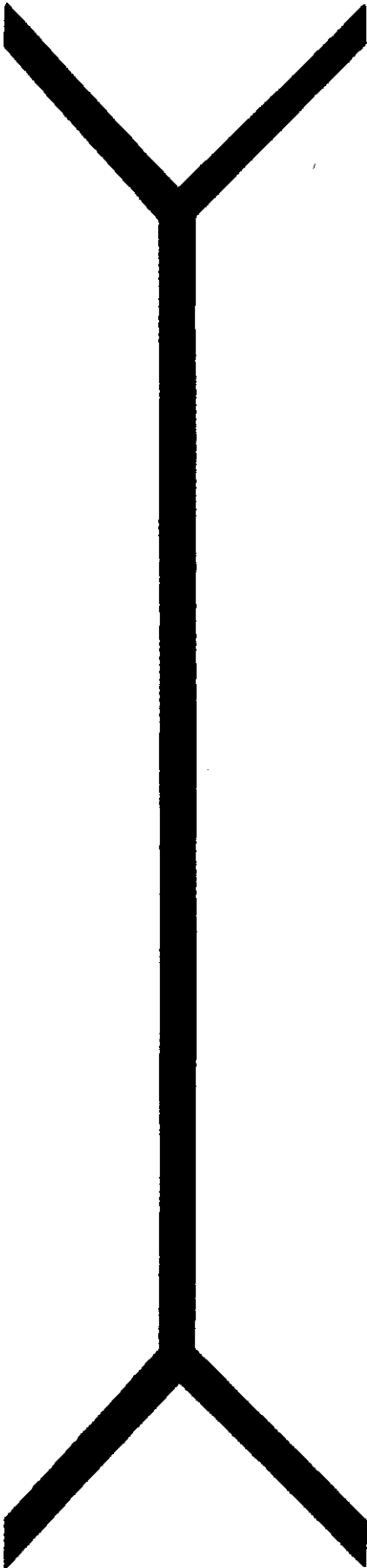
Do these lines meet up – or not?



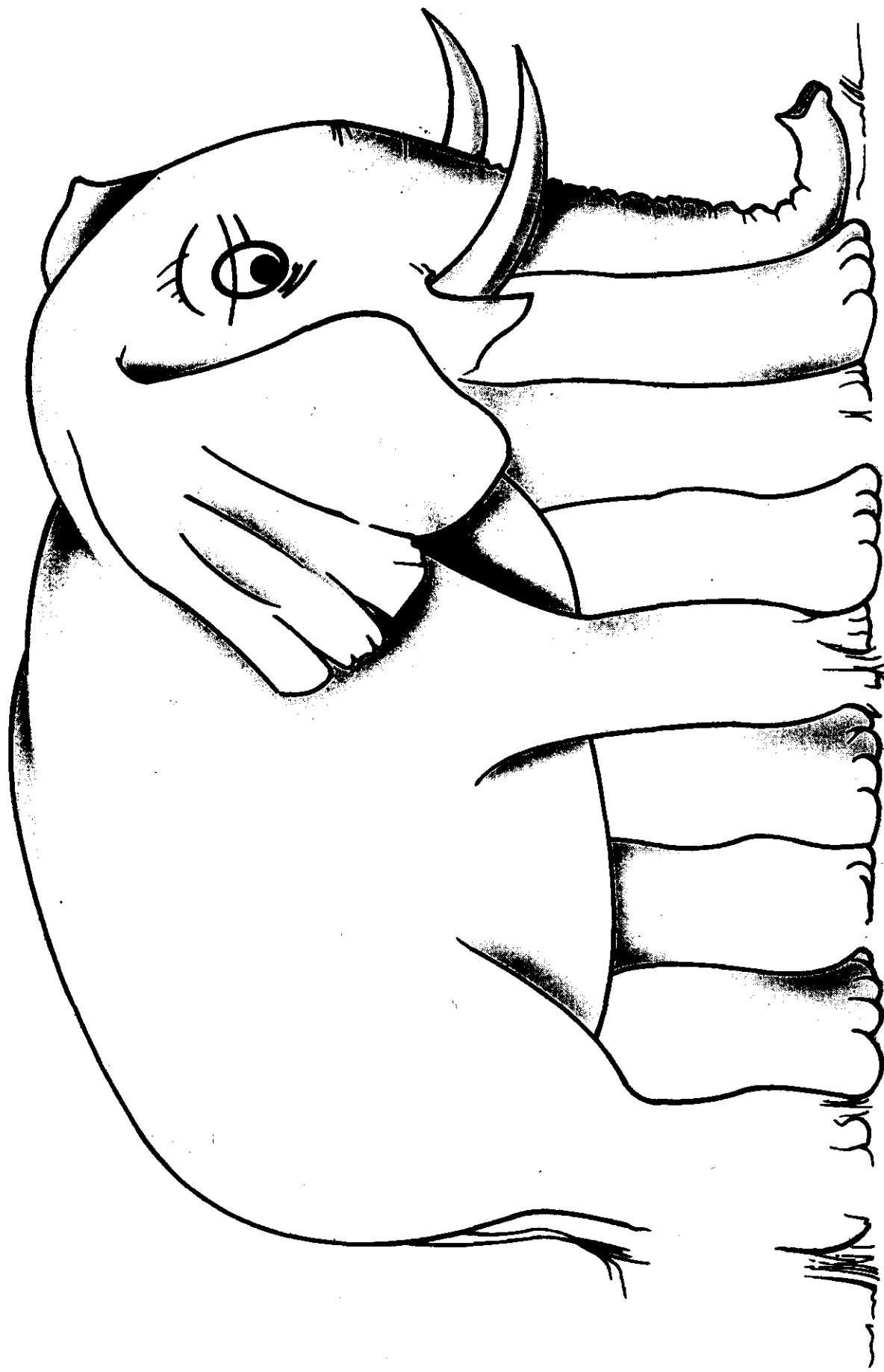
Are these lines all the same length?

Are the dots different distances apart?



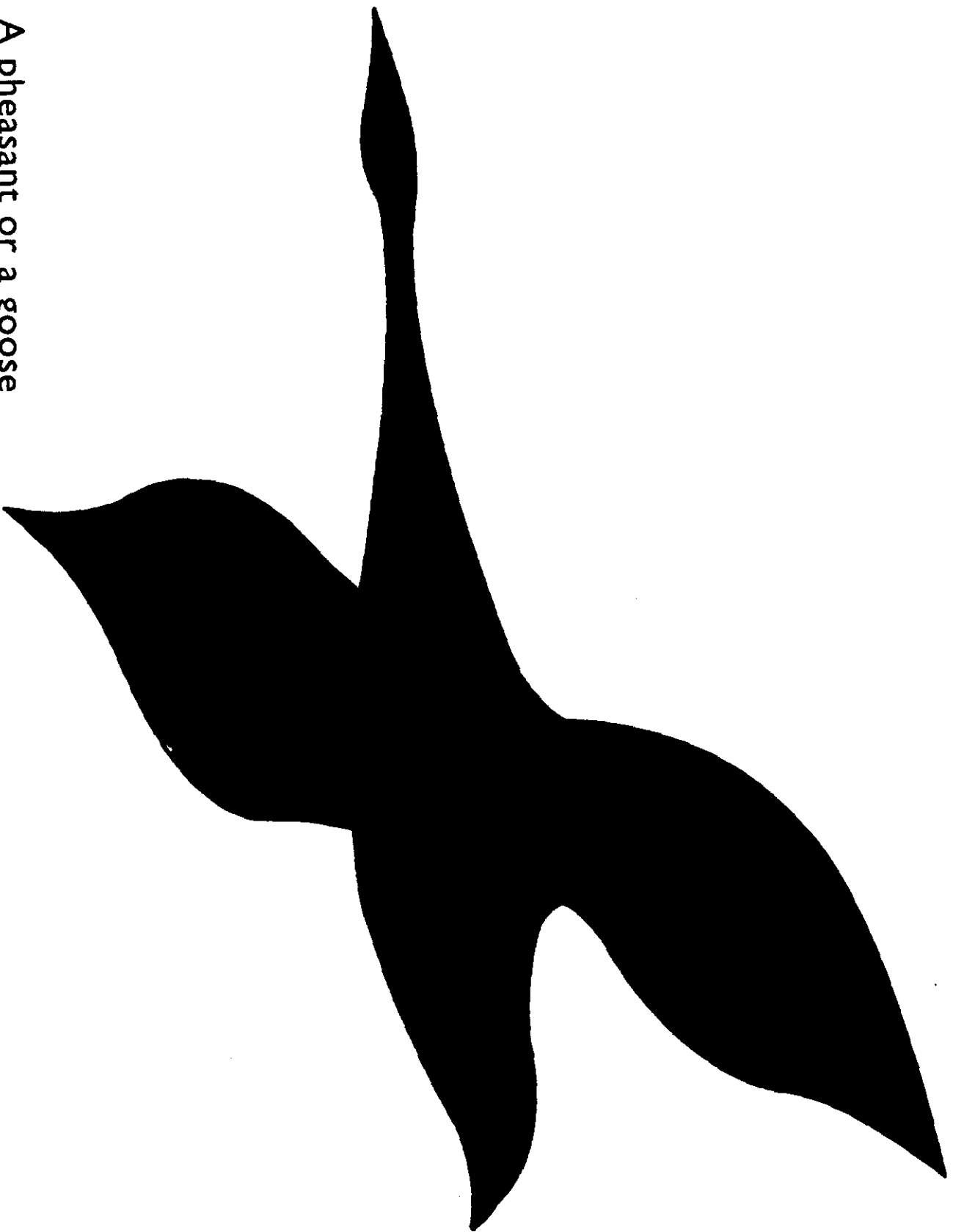


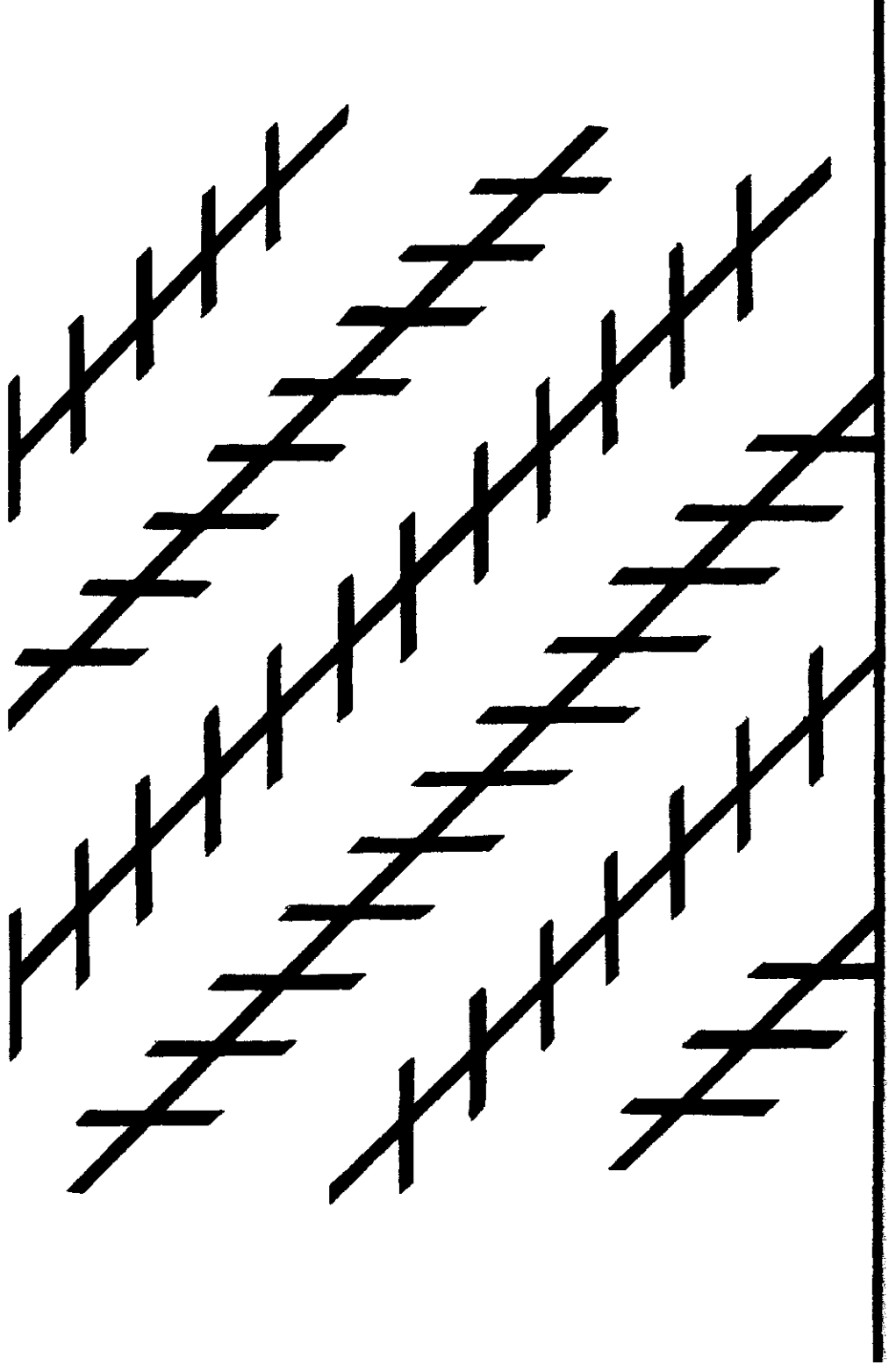
Which line is longer?



Can this elephant walk?

A pheasant or a goose





Are these lines parallel? Try changing your viewpoint!

LINES

- Three Make A Line
 - explores the concept of three points defining a line
- Straight Lines Dividing A Plane And Planes Dividing A Region
 - explores a plane divided by lines into regions and creating a formula for the pattern
 - explores a region divided by planes

LINES BIBLIOGRAPHY

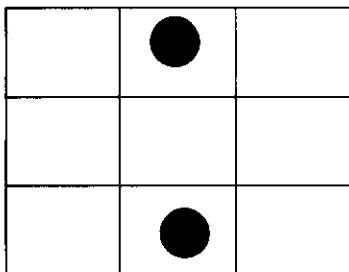
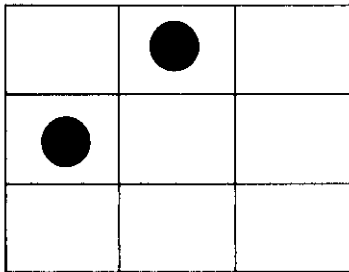
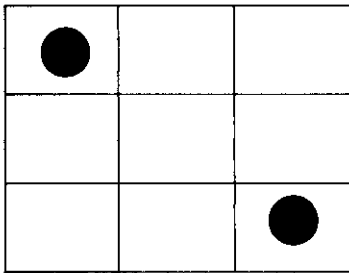
Bolt, Brian. *Mathematical Activities: A resource book for teachers*. New York, NY: Press Syndicate of the University of Cambridge, 1982.

THREE MAKE A LINE

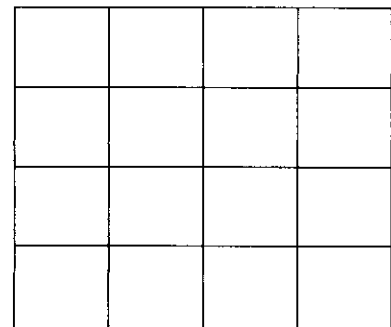
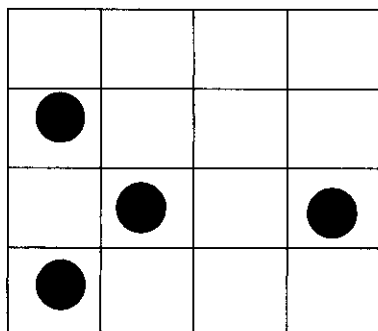
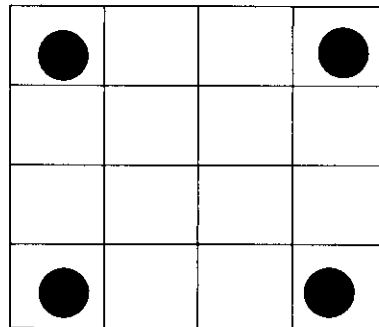
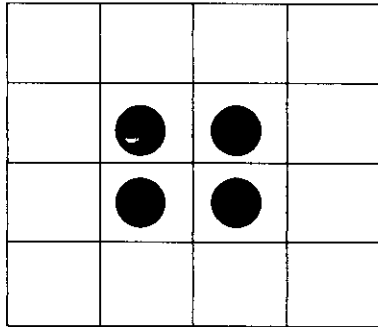
Objective: put as many counters onto the 3 x 3 board as you can so that

- i. no more than one counter is on a square
- ii. no three counters are in a straight line

To each diagram below add four more counters so that no three counters are in a line. At the right draw a diagram of how to arrange each board.

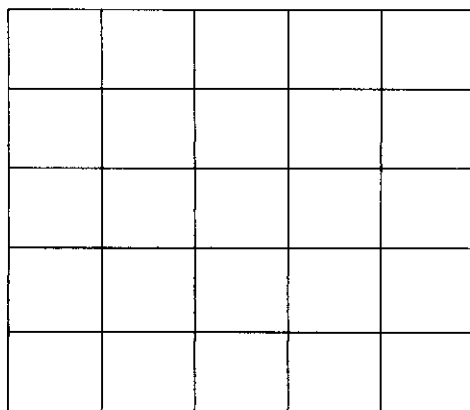


Now using the same rules from above, how many counters can be added to each board before getting three in a line?



Now find an arrangement of eight counters on a 4 x 4 board.

Now determine how to arrange 10 counters on a 5 x 5 board.



THREE MAKE A LINE - ANSWER

3 x 3 Boards

Each 3 x 3 board will have the same solution.

●	●	
●		●
	●	●

4 x 4 Boards

No counters can be added to the first two boards. The third board will have 3 added counters.

	○	○	
●			
	●		●
●			○

	●	●	
●			●
●			●
	●	●	

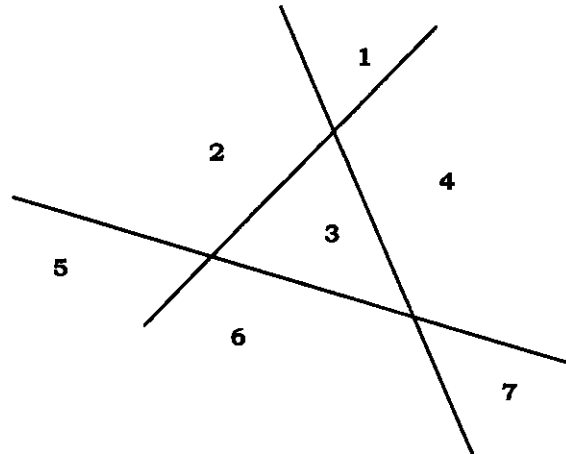
●		●	
●		●	
	●		●
	●		●

5 x 5 Boards

		●	●	
		●		●
●				●
	●		●	
●	●			

STRAIGHT LINES DIVIDING A PLANE AND PLANES DIVIDING A REGION

The diagram shows how three straight lines can be drawn in a plane so that it is divided into at most seven regions.



Complete the table below showing the maximum number of regions that can be formed in each case.

Number of lines (n)	0	1	2	3	4	5	6	7
Number of regions (r)				7				

By continuing the sequence without drawing lines, can you tell how many regions can be formed with

- a.) 10 lines?
- b.) 100 lines?

Going further...

Try to determine the maximum number of regions which three dimensional space can be divided into by intersecting planes!

STRAIGHT LINES DIVIDING A PLANE AND PLANES DIVIDING A REGION - ANSWER

Number of lines (n)	0	1	2	3	4	5	6	7
Number of regions (r)	1	2	4	7	11	16	22	29

By continuing the sequence without drawing lines can you tell how many regions can be formed with

a.) 10 lines? (**r = 56**)

b.) 100 lines? (**r = 5051**)

To determine the pattern, look at the differences between the “r” values (1, 2, 3, 4, 5,...). From this a formula for “r” can be created.

$$r = 1 + \frac{1}{2} n(n + 1)$$

Going further...

Try to determine the maximum number of regions which three dimensional space can be divided into by intersecting planes!

Number of planes (p)	0	1	2	3	4
Number of regions (r)	1	2	4	8	15

	1	2	4	8	15	26	42
1 st difference		1	2	4	7	11	16
2 nd difference			1	2	3	4	5

TRIANGLES

- Center Of A Triangle Using String
 - explores the concept that the “center” of a triangle can be determined in multiple ways
- How Many Triangles Can You See?
 - exposes students to finding embedded figures
- Triangles On A Pin Board
 - explore area of triangles
 - recognizing possible triangles
- Toothpick Triangles
 - explores the idea of equilateral triangles
- Angles Of Triangles
 - uses properties of angle measure
- Advanced Angles of Triangles
 - uses properties of angle measure
- The Big Triangle Problem
 - uses properties of angle measure
- Medians Of Triangles
 - using the Midpoint Formula
- The Vocabulary Of Triangles
 - familiarize students to some of the definitions used with triangles
- Triangle Attributes
 - investigates angles in triangles
- Sides Of A Triangle
 - Pythagorean theorem
 - Triangle Inequality Theorem
- Special Right Triangles
 - $30^{\circ} - 60^{\circ} - 90^{\circ}$ triangles
 - $45^{\circ} - 45^{\circ} - 90^{\circ}$ triangles
- Using Trigonometric Relationships
 - explore relationships of the side lengths of triangles
- What’s Missing To Prove The Triangles Congruent?
 - using congruency theorems
- Which One Is Different?
 - applying multiple attributes of triangles

TRIANGLES BIBLIOGRAPHY

Bolt, Brian. *Mathematical Activities: A resource book for teachers*. New York, NY: Press Syndicate of the University of Cambridge, 1982.

Muschla, Judith A., and Gary R. Muschla. *Geometry Teacher's Activities Kit: Ready-to-Use Lessons and Worksheets for Grades 6 – 12*. West Nyack, NY: The Center For Applied Research In Education, 2000.

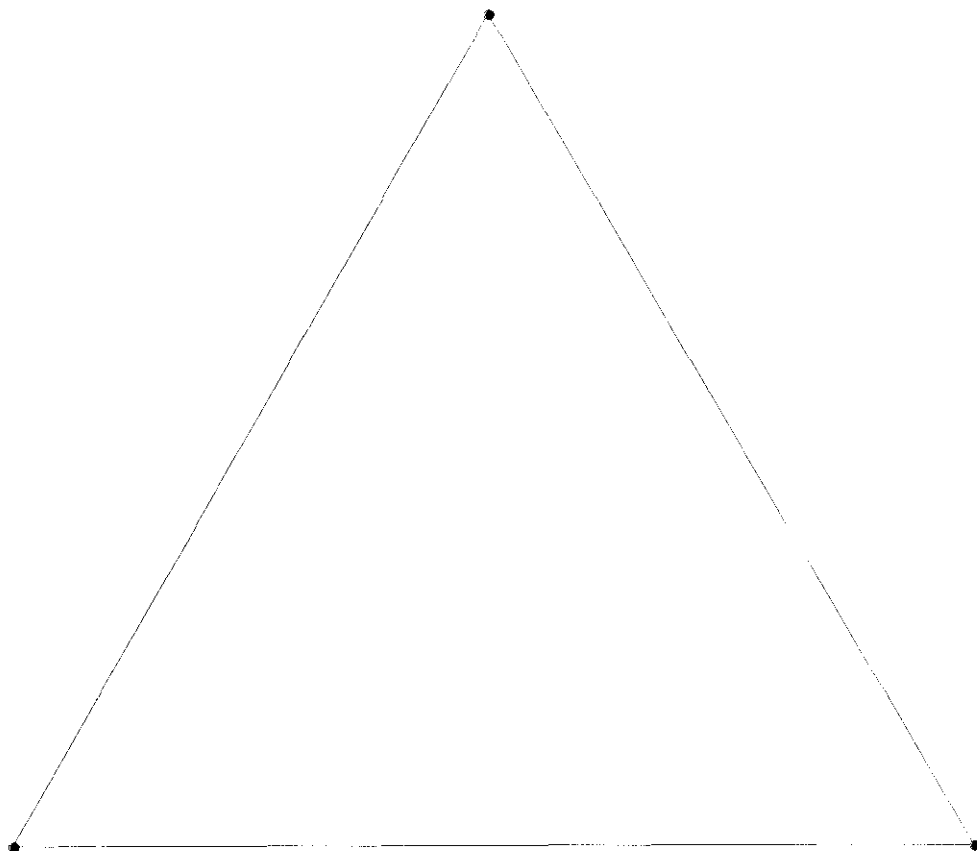
Pohl, Victoria. *How to Enrich Geometry Using String Designs*. Reston, VA: The National Council Of Teachers Of Mathematics, Inc., 1995.

CENTER OF A TRIANGLE USING STRING

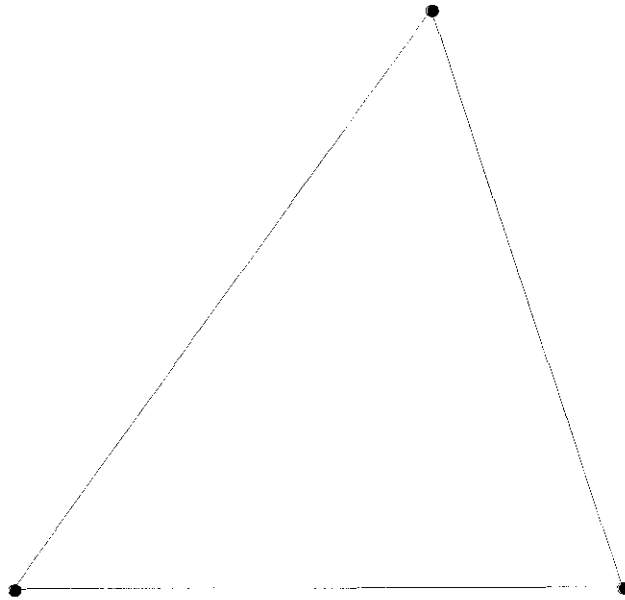
Using the provided triangles, follow the directions to find the centroid, incenter, orthocenter and circumcenter.

1. Construct the medians of the triangle.
2. Label the midpoint of each side with a “ ! ”.
3. Bisect each angle of the triangle, extending the angle bisector so that it crosses the opposite side of the triangle.
4. Label each intersection of the angle bisector and side with a “ \$ ”.
5. Construct the altitude for each side of the triangle.
6. Mark the intersection of the altitude with the side with a “ @ ”.
7. Construct the perpendicular bisectors of each side of the triangle.
8. Mark the intersection of the perpendicular bisector and the side of the triangle with a “ # ”.
9. Cut out a square around the triangle.
10. Attach your triangle to a piece of card stock or similar paper. (The side with the triangle will be considered the “back” and the side without the triangle will be considered the “front.”)
11. Take a needle and make a hole at each intersection point marked with a symbol on an edge of the triangle and the vertices.
12. Take black thread and connect the vertices so that an outline of the triangle is formed in string on the front side of the cardstock.
13. Choose a color of string for each symbol.
!
\$
@
#
14. Looking at the triangle, start at an intersection point marked by the ! and connect this point to its respective vertex. You should have a string line on the front side of the cardstock. Connect the other two intersection points marked with a ! to their respective vertices using the same color of string.
15. Now change the color of the string and repeat the above process with the next symbol until all four symbols are done.
16. Looking at the front side of the triangle, determine where each color of string intersects.
17. Make observations concerning the “center” of a triangle.

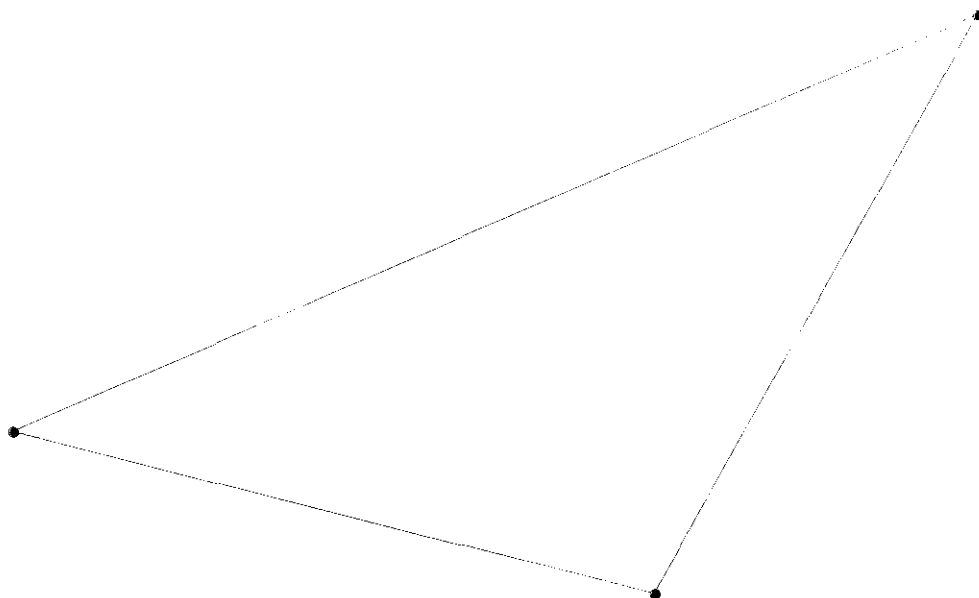
EQUILATERAL TRIANGLE



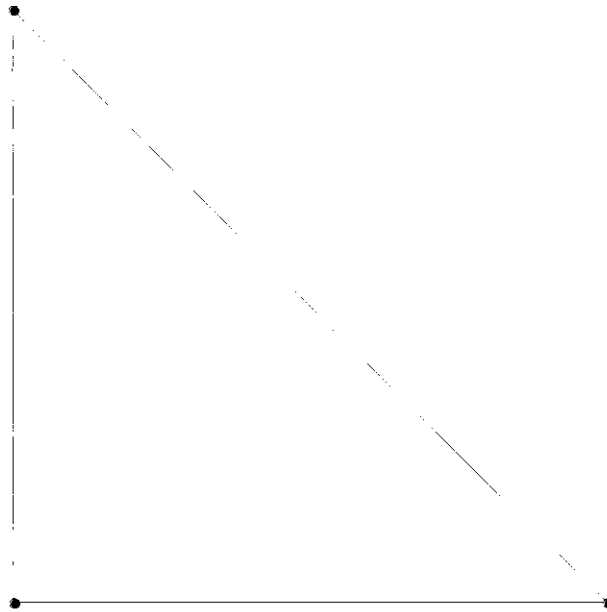
ACUTE ISOSCELES



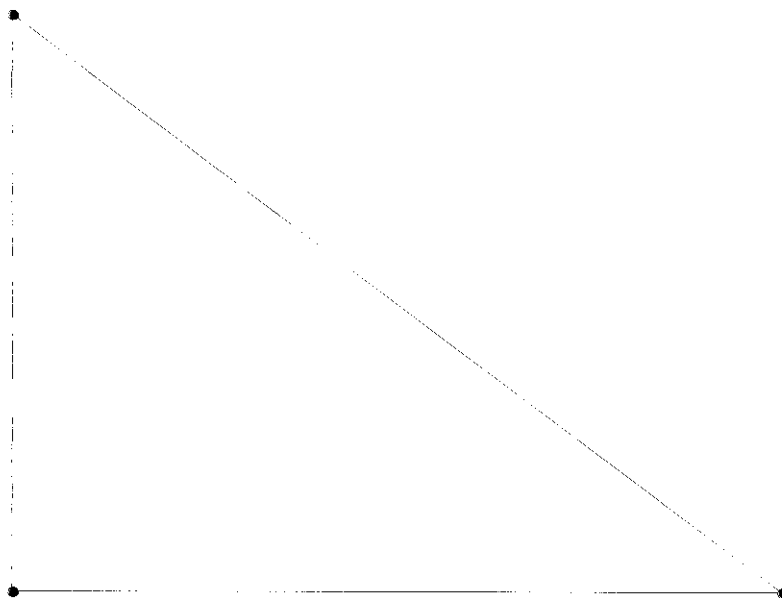
OBTUSE ISOSCELES



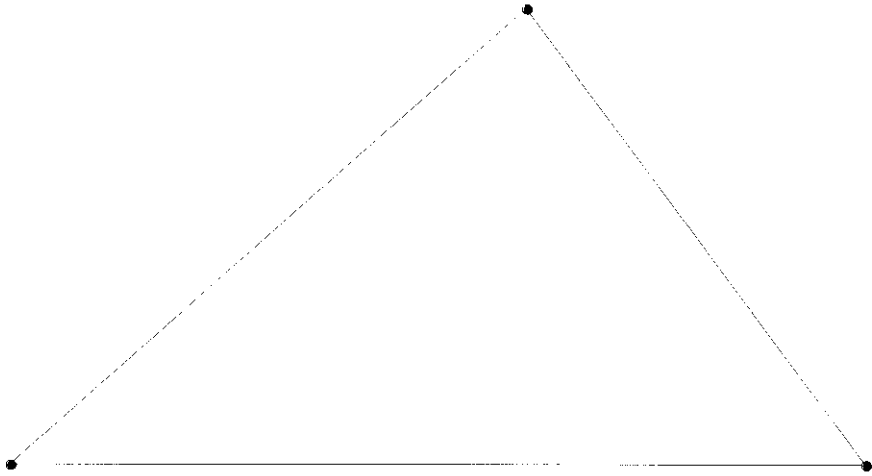
RIGHT ISOSCELES



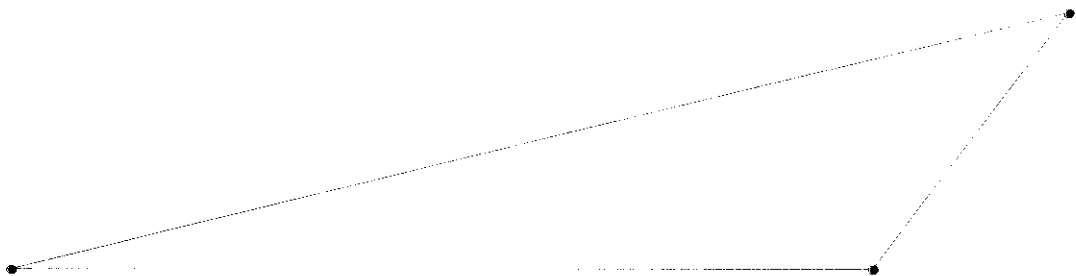
RIGHT SCALENE



ACUTE SCALENE

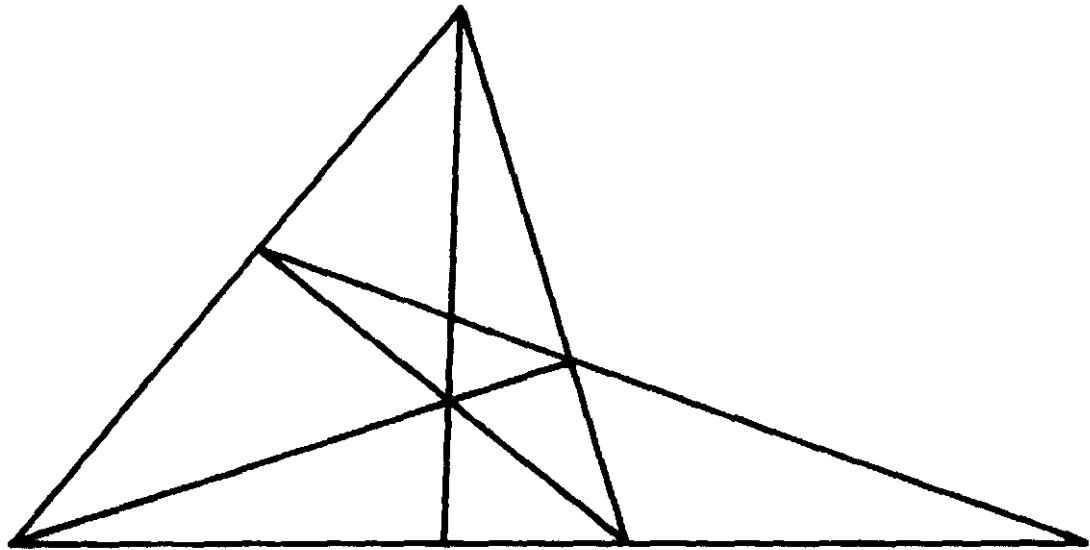


OBTUSE SCALENE



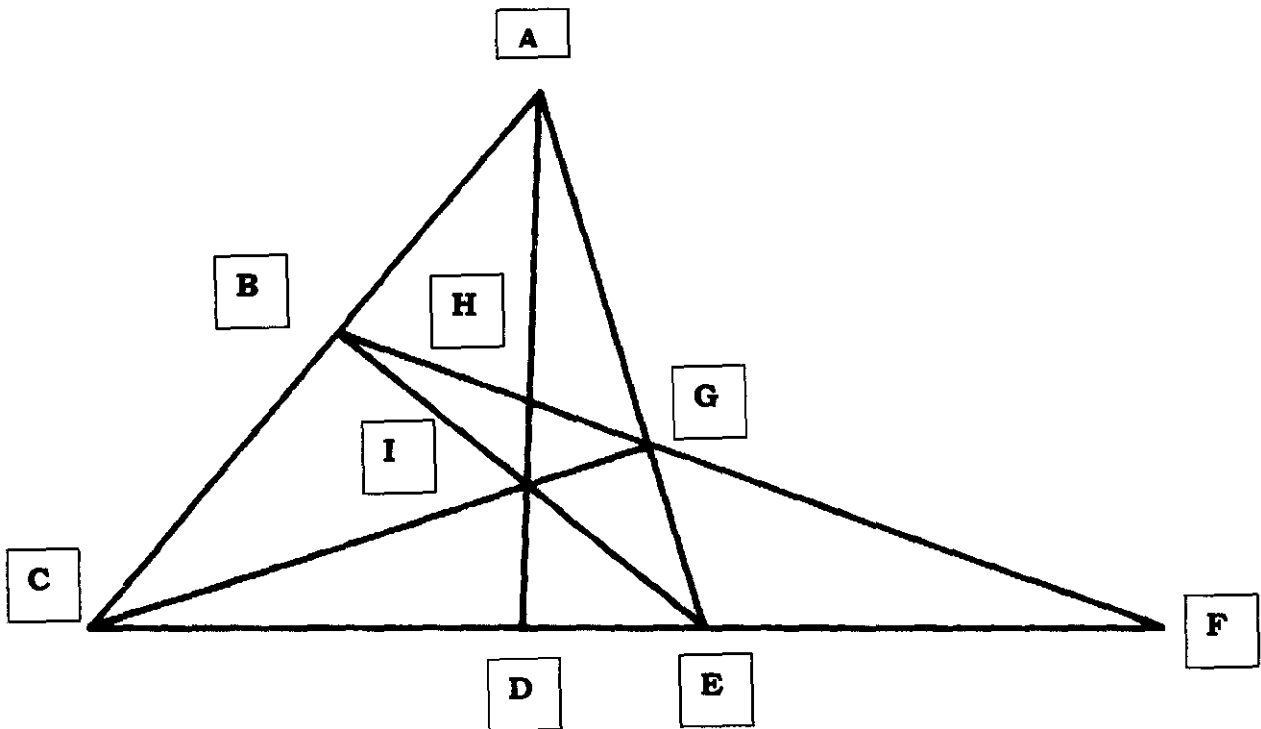
HOW MANY TRIANGLES CAN YOU SEE?

The figure contains many triangles, some of which overlap each other.
Find a systematic way of accounting for all the triangles.



HOW MANY TRIANGLES? - ANSWER

Label the vertices, starting at the top and working around the triangles counter clockwise.

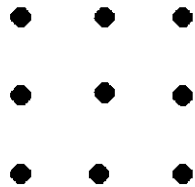


There are 29 triangles in all.

ABE	ABG	ABH	ABI
ACD	ACE	ACG	ACI
ADE	AEI	AGH	AGI
BCE	BCF	BCG	BCI
BEF	BEG	BGI	BHI
CDI	CEG	CEI	CFG
DEI	DFH	EFG	EGI
GHI			

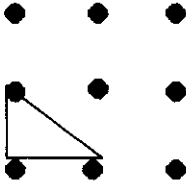
TRIANGLES ON A PIN BOARD

How many triangles can be made on a 3 x 3 pin board?

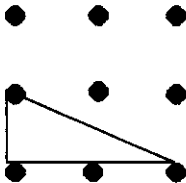


TRIANGLES ON A PIN BOARD - ANSWERS

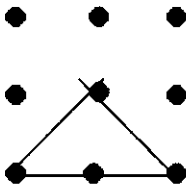
There are 8 types of triangles that can be formed. In all there are 76 different triangles that can be formed.



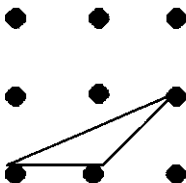
There are 16 of these types of triangles possible.



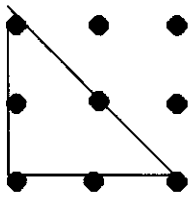
There are 16 of these types of triangles possible.



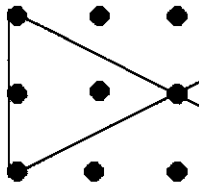
There are 8 of these types of triangles possible.



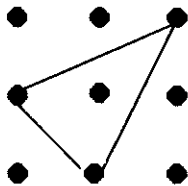
There are 16 of these types of triangles possible.



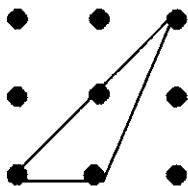
There are 4 of these types of triangles possible.



There are 4 of these types of triangles possible.



There are 4 of these types of triangles possible.



There are 8 of these types of triangles possible.

TOOTHPICK TRIANGLES

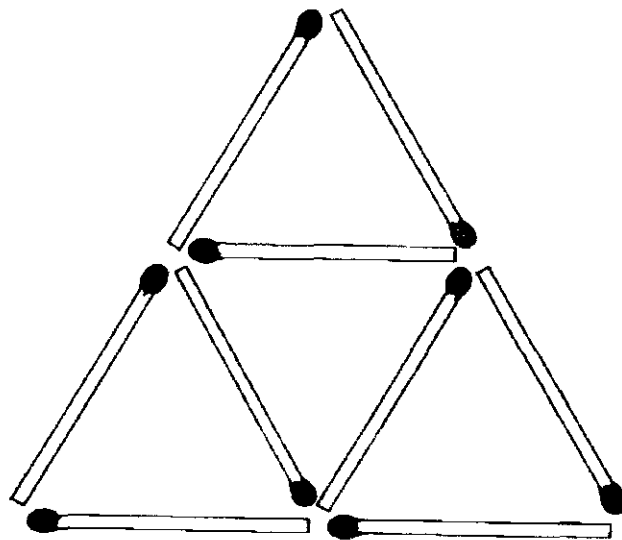
Arrange 9 toothpicks to form four triangles.

What kind of triangles will be formed?

How do you know?

TOOTHPICK TRIANGLES – ANSWERS

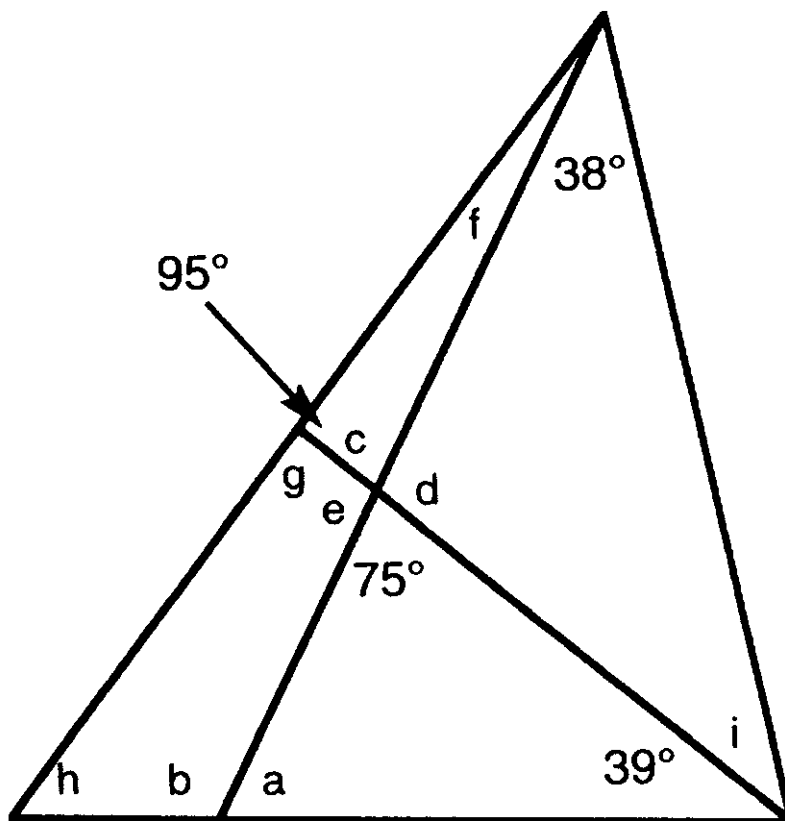
The four triangles that will be formed are equilateral triangles because each side will be formed using equal lengths.



ANGLES OF TRIANGLES

Using the following three facts about triangles determine the measures of angles a through i in the figure. Tell the measure of each angle and provide a reason.

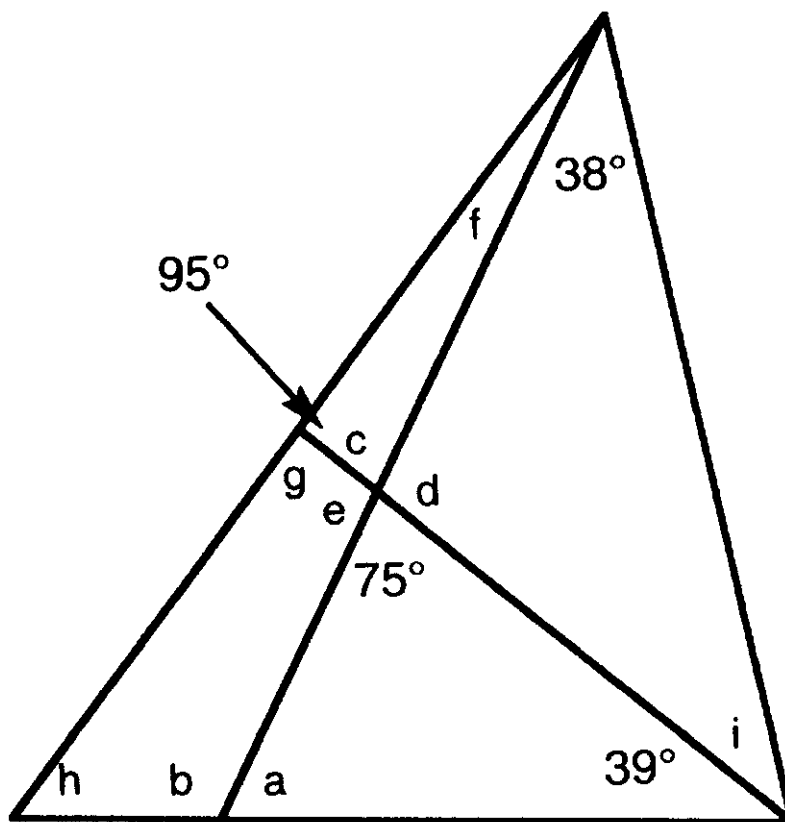
- The sum of the measures of the angles in a triangle equals 180° .
- Vertical angles are congruent.
- The measure of a straight angle is 180° .



ANGLES OF TRIANGLES – ANSWER

Using the following three facts about triangles determine the measures of angles *a* through *i* in the figure. Tell the measure of each angle and provide a reason.

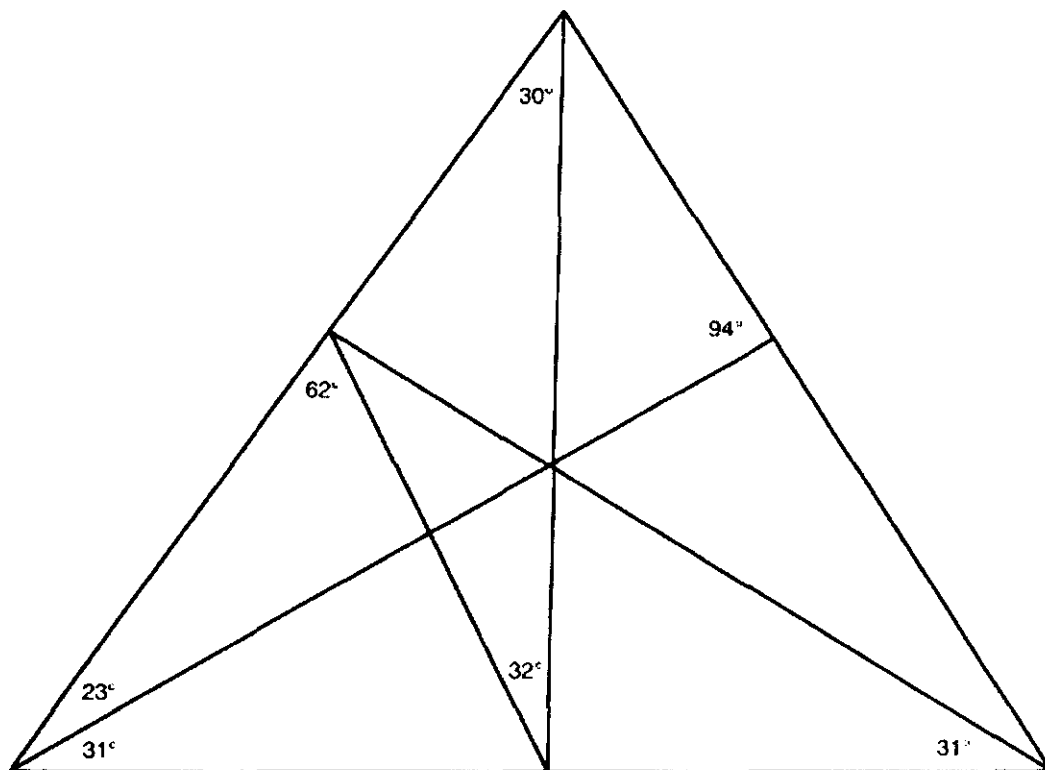
- The sum of the measures of the angles in a triangle equals 180° .
- Vertical angles are congruent.
- The measure of a straight angle is 180° .



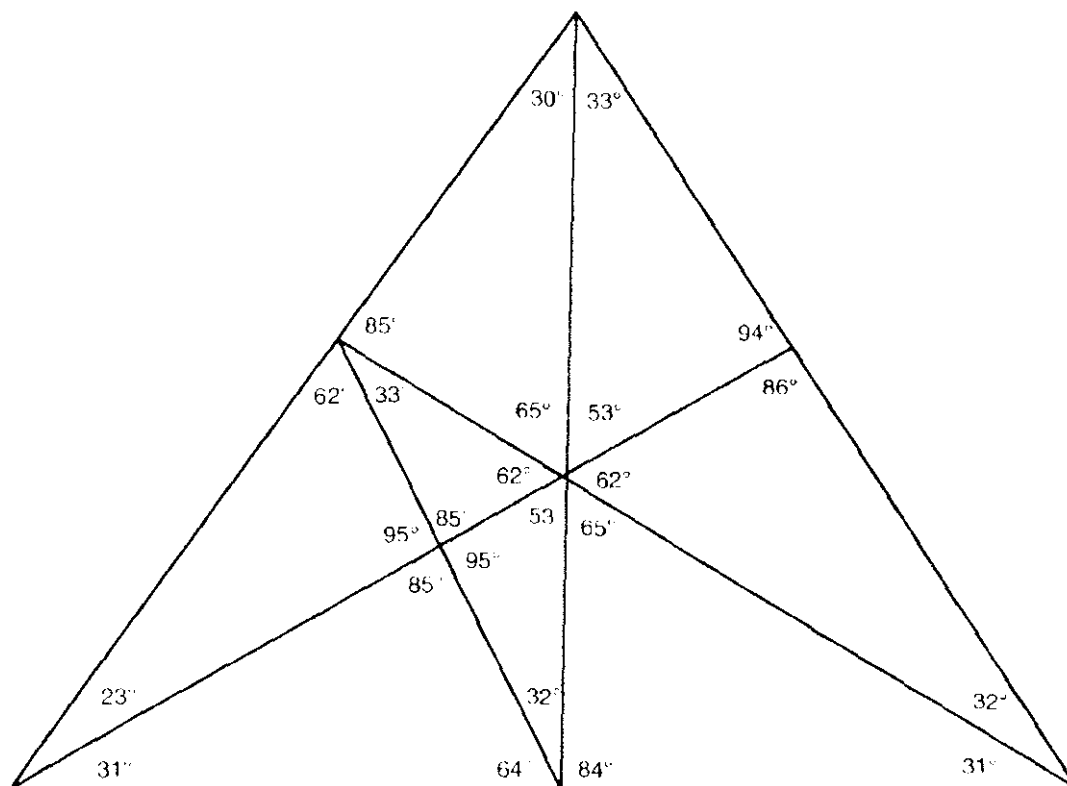
a. 66° ; b. 114° ; c. 75° ; d. 105° ; e. 105° ; f. 10° ; g. 85° ; h. 56° ; i. 37°

ADVANCED ANGLES OF TRIANGLES

Fill in the missing angle measures in the figure below. (Do not use a protractor!)



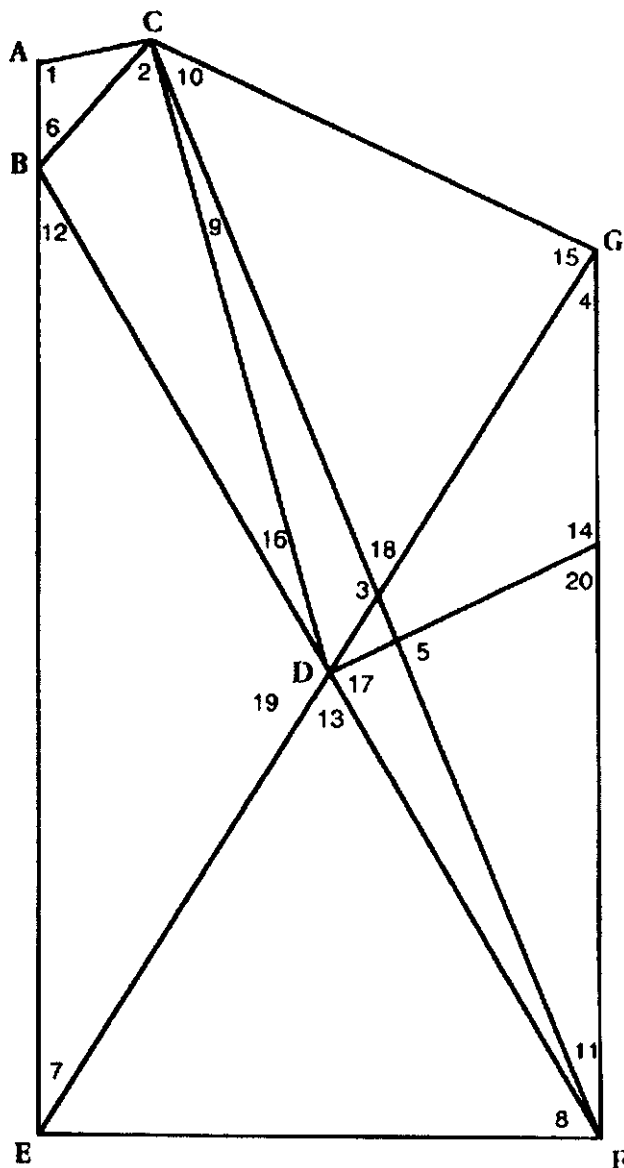
ADVANCED ANGLES OF TRIANGLES - ANSWER



THE BIG TRIANGLE PROBLEM

Using the given information find the missing angle measures.

- $\triangle ABC$ is isosceles with base \overline{BC} .
- $\overline{AE}, \overline{EG}, \overline{BF},$ and \overline{CF} are line segments.
- $m\angle 1 = 100$, $m\angle 2 = 56$, $m\angle 3 = 128$, $m\angle 4 = 36$, $m\angle 5 = 94$
- $\triangle DEF$ is equilateral
- $AE \perp EF$, $CG \perp EG$.



THE BIG TRIANGLE PROBLEM - ANSWER

Angle 6 : 40
Angle 7: 30
Angle 8: 60
Angle 9: 6
Angle 10: 38
Angle 11: 16
Angle 12: 30
Angle 13: 60
Angle 14: 110
Angle 15: 90
Angle 16: 14
Angle 17: 86
Angle 18: 52
Angle 19: 120
Angle 20: 70

MEDIANS OF TRIANGLES

The median of a triangle is a segment drawn from a vertex of a triangle to the midpoint of a nonadjacent side. In this activity you will graph the vertices of a triangle and connect the vertices to form the triangle. Then you will use the Midpoint Formula to draw the medians.

Midpoint Formula: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$, where (x_1, y_1) and (x_2, y_2) are endpoints of a segment.

Example: the endpoints of a segment are (8, 3) and (-2, 4), then the midpoint is $\left(\frac{8 + -2}{2}, \frac{4 + 3}{2} \right)$ or (3, 3.5).

For each of the four sets of points below, plot the points, label the coordinates, and connect the points to form a triangle. Select the endpoints of each segment and use the Midpoint Formula to find the coordinates of the midpoint of the segment. To draw the median, connect the midpoint to the opposite vertex. Record the midpoint of each segment on the graph.

1. (1, 2), (7, 11), (13, 2)
2. (-10, 0), (-10, 9), (5, 9)
3. (2, -1), (-3, -7), (-10, -5)
4. (7, 1), (12, -2), (3, -7)

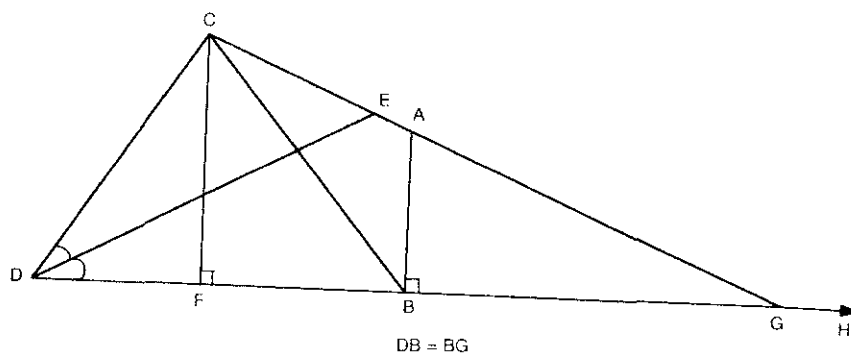
MEDIANS OF TRIANGLES - ANSWER

Coordinates of the midpoints:

1. $(4, 6.5), (10, 6.5), (7, 2)$
2. $(-10, 4.5), (-2.5, 9), (-2.5, 4.5)$
3. $(-.5, -4), (-6.5, -6), (-4, -3)$
4. $(9.5, -.5), (7.5, -4.5), (5, -3)$

THE VOCABULARY OF TRIANGLES

The diagram below contains definitions and symbols associated with triangles. For each line in the left column find the appropriate description in the right column. When you are finished, reading down the left will reveal a message.



- | | |
|---|---|
| <p>_____ $\triangle ABG$</p> <p>_____ B</p> <p>_____ $\angle CGH$</p> <p>_____ $\angle GCD$</p> <p>_____ $\angle CFD$</p> <p>_____ $\angle CDE$ and $\angle EDG$</p> <p>_____ $\angle BAG$ and $\angle AGB$</p> <p>_____ $\angle CAB$ and $\angle BAG$</p> <p>_____ \overline{CF}</p> <p>_____ \overline{AB}</p> <p>_____ \overline{CB}</p> <p>_____ \overline{DE}</p> <p>_____ \overline{AG}</p> <p>_____ CF</p> <p>_____ \overline{CF} and \overline{AB}</p> <p>_____ \overline{CF} and \overline{DG}</p> <p>_____ \overline{AB} and \overline{BG}</p> | <p>Y. a right angle</p> <p>T. complementary angles</p> <p>I. an altitude</p> <p>Y. a perpendicular bisector</p> <p>U. an angle bisector</p> <p>A. a right triangle</p> <p>R. a hypotenuse</p> <p>E. parallel segments</p> <p>S. perpendicular segments</p> <p>W. an exterior angle</p> <p>R. supplementary angles</p> <p>B. length of the altitude</p> <p>L. a midpoint</p> <p>O. a median</p> <p>A. a remote interior angle of $\angle CGH$</p> <p>S. congruent angles</p> <p>T. legs of a right triangle</p> |
|---|---|

THE VOCABULARY OF TRIANGLES – ANSWERS

Always tri your best.

TRIANGLE ATTRIBUTES

Draw the triangles listed in the left column of the table below then decide if the triangle “must”, “may”, or “can’t” have each characteristic.

Types of Triangles	1 right angle	1 obtuse angle	2 acute angles	3 acute angles	2 congruent angles	3 congruent angles	2 congruent sides
Right							
Acute							
Obtuse							
Scalene							
Isosceles							
Equilateral							
Right Isosceles							
Right Scalene							

TRIANGLE ATTRIBUTES – ANSWERS

Draw the triangles listed in the left column of the table below then decide if the triangle “must”, “may”, or “can’t” have each characteristic.

Types of Triangles	1 right angle	1 obtuse angle	2 acute angles	3 acute angles	2 congruent angles	3 congruent angles	2 congruent sides
Right	<i>Must</i>	<i>Can't</i>	<i>Must</i>	<i>Can't</i>	<i>May</i>	<i>Can't</i>	<i>May</i>
Acute	<i>Can't</i>	<i>Can't</i>	<i>Must</i>	<i>Must</i>	<i>May</i>	<i>May</i>	<i>May</i>
Obtuse	<i>Can't</i>	<i>Must</i>	<i>Must</i>	<i>Can't</i>	<i>May</i>	<i>Can't</i>	<i>May</i>
Scalene	<i>May</i>	<i>May</i>	<i>Must</i>	<i>May</i>	<i>Can't</i>	<i>Can't</i>	<i>Can't</i>
Isosceles	<i>May</i>	<i>May</i>	<i>Must</i>	<i>May</i>	<i>Must</i>	<i>May</i>	<i>Must</i>
Equilateral	<i>Can't</i>	<i>Can't</i>	<i>Must</i>	<i>Must</i>	<i>Must</i>	<i>Must</i>	<i>Must</i>
Right Isosceles	<i>Must</i>	<i>Can't</i>	<i>Must</i>	<i>Can't</i>	<i>Must</i>	<i>Can't</i>	<i>Must</i>
Right Scalene	<i>Must</i>	<i>Can't</i>	<i>Must</i>	<i>Can't</i>	<i>Can't</i>	<i>Can't</i>	<i>Can't</i>

SIDES OF A TRIANGLE

The *Triangle Inequality Theorem* says the sum of the lengths of any two sides of a triangle is greater than the length of the third side. Using this theorem with the Pythagorean theorem you can determine an unknown third side of a triangle to meet specific qualifications.

Relationship Between the Sides of the Triangle	Type of Triangle
$a^2 + b^2 = c^2$	right
$a^2 + b^2 > c^2$	acute
$a^2 + b^2 < c^2$	obtuse

For the following problem use the above information to determine a possible length for the longest side of the triangle. (There is a range of answers for some of the triangles.)

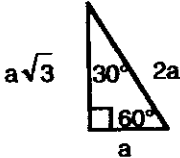
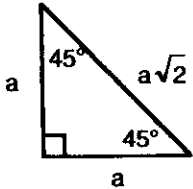
1. Obtuse triangle: Side lengths of 5, 8, and _____
2. Acute triangle: Side lengths of 5, 6, and _____
3. Right triangle: Side lengths of 5, 12, and _____
4. Acute triangle: Side lengths of 2, 2, and _____
5. Right triangle: Side lengths of 10, 24, and _____
6. Obtuse triangle: Side lengths of 3, 4, and _____
7. Acute triangle: Side lengths of 5, 12, and _____
8. Right triangle: Side lengths of 7, 24, and _____

SIDES OF A TRIANGLE – ANSWERS

(c represents the longest side length of the triangle)

1. Obtuse triangle: Side lengths of 5, 8, and $\sqrt{89} < c < 13$
2. Acute triangle: Side lengths of 5, 6, and $6 \leq c < \sqrt{61}$
3. Right triangle: Side lengths of 5, 12, and $c = 13$
4. Acute triangle: Side lengths of 2, 2, and $2 \leq c < 2\sqrt{2}$
5. Right triangle: Side lengths of 10, 24, and $c = 26$
6. Obtuse triangle: Side lengths of 3, 4, and $5 < c < 7$
7. Acute triangle: Side lengths of 5, 12, and $12 \leq c < 13$
8. Right triangle: Side lengths of 7, 24, and $c = 25$

SPECIAL RIGHT TRIANGLES

<p style="text-align: center;">30°-60°-90° triangle</p> 	<p>The length of the hypotenuse is twice the length of the shorter leg. The length of the longer leg is $\sqrt{3}$ times the length of the shorter leg.</p>
<p style="text-align: center;">45°-45°-90° triangle</p> 	<p>The two legs are congruent. The length of the hypotenuse is equal to the length of a leg times $\sqrt{2}$.</p>

Use the following information about the triangles to determine if the triangle has one of the two special relationships or if it is simply a right triangle.

1. The hypotenuse is $5\sqrt{2}$. Both legs are congruent.
2. The longer leg is 6. The hypotenuse is 8.
3. The shorter leg is 3. The longer leg is 6.
4. The shorter leg is 12. The hypotenuse is 24.
5. The longer leg is $5\sqrt{3}$. The hypotenuse is 10.
6. The hypotenuse is 3. Both legs are congruent.
7. The shorter leg is 3. The hypotenuse is 5.
8. The legs are congruent. Each equals 3.
9. The longer leg is 6. The hypotenuse is $4\sqrt{3}$.
10. The shorter leg is 6. The hypotenuse is 12.

SPECIAL RIGHT TRIANGLES - ANSWERS

1. each leg = 5 ; 45 – 45 – 90 triangle
2. shorter leg = $2\sqrt{7}$; right triangle
3. hypotenuse = $3\sqrt{5}$; right triangle
4. longer leg = $12\sqrt{3}$; 30 – 60 – 90 triangle
5. shorter leg = 5 ; 30 – 60 – 90 triangle
6. each leg = $\frac{3\sqrt{2}}{2}$; 45 – 45 – 90 triangle
7. longer leg = 4 ; right triangle
8. hypotenuse = $3\sqrt{2}$; 45 – 45 – 90 triangle
9. shorter leg = $2\sqrt{3}$; 30 – 60 – 90
10. longer leg = $6\sqrt{3}$; 30 – 60 – 90 triangle

USING TRIGONOMETRIC RELATIONSHIPS

$$\sin a = \frac{\textit{opposite}}{\textit{hypotenuse}} \quad \cos a = \frac{\textit{adjacent}}{\textit{hypotenuse}} \quad \tan a = \frac{\textit{opposite}}{\textit{adjacent}}$$

Use the trigonometric relationships and special triangles to explain the following.

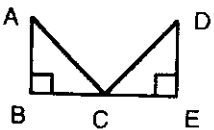
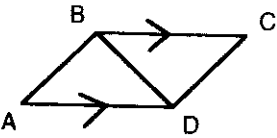
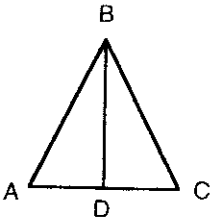
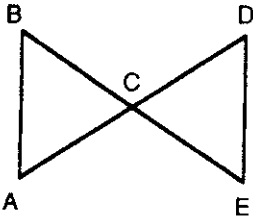
STATEMENT	REASON
The tangent of 90° is undefined.	
The sine of 30° is 0.5.	
The cosine of 60° is 0.5.	
The tangent of $60^\circ = \sqrt{3}$.	
The tangent of $45^\circ = 1$.	
The sine of 45° and the cosine of 45° are the same ratios.	
The sine of the measure of an angle is equal to the cosine of 90° minus the measure of the angle.	
The sine of the measure of any acute angle in a right triangle is less than 1.	
The tangent of the measure of any acute angle larger than 45° is greater than 1.	
A “small” angle has a sine and tangent ratios that are both very small.	

USING TRIGONOMETRIC RELATIONSHIPS - ANSWERS

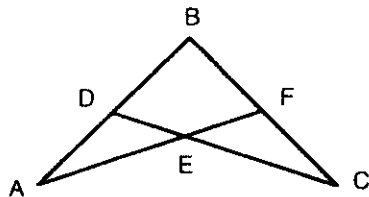
STATEMENT	REASON
The tangent of 90° is undefined.	<i>The ratios are defined for the acute angles of a right triangle.</i>
The sine of 30° is 0.5.	$\sin 30^\circ = \frac{a}{2a} = \frac{1}{2}$
The cosine of 60° is 0.5.	$\cos 60^\circ = \frac{a}{2a} = \frac{1}{2}$
The tangent of $60^\circ = \sqrt{3}$.	$\tan 60^\circ = \frac{a\sqrt{3}}{a} = \sqrt{3}$
The tangent of $45^\circ = 1$.	$\tan 45^\circ = \frac{a}{a} = 1$
The sine of 45° and the cosine of 45° are the same ratios.	<i>The length of the opposite leg and the adjacent leg are the same.</i>
The sine of the measure of an angle is equal to the cosine of 90° minus the measure of the angle.	<i>The two acute angles of a right triangle are complimentary.</i>
The sine of the measure of any acute angle in a right triangle is less than 1.	<i>The length of any side of a right triangle is less than the length of the hypotenuse.</i>
The tangent of the measure of any acute angle larger than 45° is greater than 1.	<i>The length of the leg opposite an acute angle larger than 45° is greater than the length of the other leg.</i>
A "small" angle has a sine and tangent ratios that are both very small.	<i>The length of the leg opposite the "small" angle is very short in relation to the hypotenuse. Therefore, the sine ratio is small. The length of the leg opposite the angle is also short in relation to the leg adjacent to it. Therefore the tangent ratio is small.</i>

WHAT'S MISSING TO PROVE THE TRIANGLES CONGRUENT?

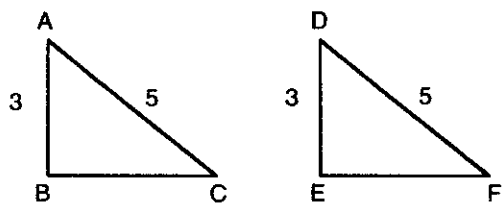
For each of the following triangles determine what information is missing to prove that the triangles are congruent.

Given Information and Diagram	Missing Information
<p>1. C is the midpoint of \overline{BE}. Prove $\triangle ABC \cong \triangle DEC$ by HL.</p> 	
<p>2. Prove $\triangle ABD \cong \triangle CDB$ by SAS.</p> 	
<p>3. $\triangle ABC$ is isosceles. $\angle A$ and $\angle C$ are base angles. Prove $\triangle ABD \cong \triangle CBD$ by ASA.</p> 	
<p>4. C is the midpoint of \overline{AD}. Prove that $\triangle ABC \cong \triangle DEC$ by ASA.</p> 	

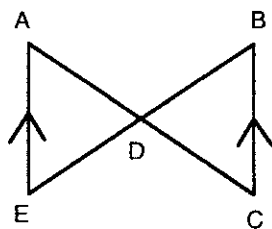
5. $\angle A \cong \angle C$. Prove $\triangle ADE \cong \triangle CFE$ by AAS.



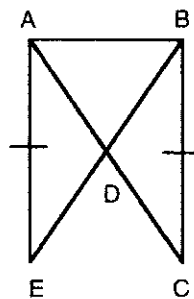
6. Prove $\triangle ABC \cong \triangle DEF$ by HL.



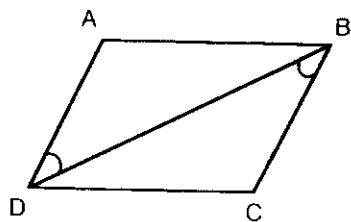
7. Prove $\triangle ADE \cong \triangle CDB$ by ASA.



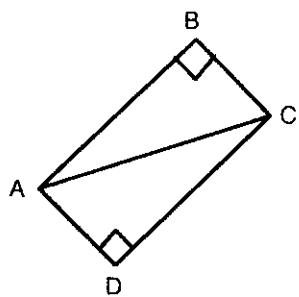
8. Prove $\triangle EAB \cong \triangle CBA$ by SSS.



9. Prove $\triangle ABD \cong \triangle CDB$ by SAS.



10. Prove $\triangle ABC \cong \triangle CDA$ by HL.



**WHAT'S MISSING TO PROVE THE TRIANGLES CONGRUENT? -
ANSWERS**

1. $\overline{AC} \cong \overline{DC}$
2. $\overline{AD} \cong \overline{CB}$
3. $\angle ABD \cong \angle CBD$
4. $\angle A \cong \angle D$
5. $\overline{AD} \cong \overline{CF}$ or $\overline{ED} \cong \overline{EF}$
6. $\angle B$ and $\angle E$ are right angles
7. $\overline{EA} \cong \overline{BC}$ or $\overline{ED} \cong \overline{BD}$ or $\overline{AD} \cong \overline{CD}$
8. $\overline{CA} \cong \overline{EB}$
9. $\overline{AD} \cong \overline{CB}$
10. $\overline{AB} \cong \overline{CD}$ or $\overline{BC} \cong \overline{DA}$

WHICH ONE IS DIFFERENT?

In each row below, there is one common characteristic. One of the four items does not belong. Decide which one, and then determine what must be done to it to make it belong.

Item:	A	B	C	D
1.	43°, 47°, 90°	30°, 60°, 90°	100°, 50°, 30°	15°, 80°, 90°
2.	80°, 70°, 30°	50°, 65°, 65°	50°, 80°, 50°	80°, 20°, 80°
3.	8, 9, 12	6, 7, 10	18, 21, 30	12, 14, 20
4.	2, 3, 4	8, 10, 10	3, 4, 7	7, 9, 10
5.	AAA	SAS	SSS	AAS
6.	legs	obtuse angle	hypotenuse	Pythagorean Theorem
7.	3, 4, 5	5, 12, 13	7, 24, 25	12, 84, 85
8.	5, 8, 10	5, 6, 7	6, 7, 8	8, 10, 11

WHICH ONE IS DIFFERENT? - ANSWERS

1. All except D could be the measures of angles in a triangle. "D" should be changed to 10° , 80° , 90° .
2. All except A are measures of the angles of an isosceles triangle. "A" should be changed to 70° , 70° , 40° .
3. All except A are the sides of similar triangles. "A" should be changed to 8 , $9\frac{1}{3}$, $13\frac{1}{3}$.
4. All except C are triangles. "C" should be changed to 4 , 5 , 7 .
5. All except A are ways to prove that triangles are congruent. "A" could be changed to ASA.
6. All except B are related to right triangles. "B" should be changed to right angle.
7. All except D are sides of a right triangle. "D" should be changed to 13 , 84 , 85 .
8. All except A are sides of an acute triangle. "A" should be changed to 8 , 9 , 10 .

POLYGONS

- **Constructing A Tangram**
 - practices measuring skills
 - uses knowledge of different shapes
 - creates materials required for other activities
- **Creating Polygons From Tangrams**
 - incorporates visualization
 - reinforces basic definitions
- **Diagonals Of A Rectangle**
 - Relationships of diagonals of a rectangle
- **Concave And Convex Polygons**
 - introduction to concave and convex polygons
 - practices applying the new terminology
- **A Big Quadrilateral**
 - practices the use of the sum of the interior angles of a triangle and the sum of the interior angles of a quadrilateral
 - practices vertical angles
 - uses the knowledge of angle addition
- **A Bigger Quadrilateral**
 - practices the use of the sum of the interior angles of a triangle and the sum of the interior angles of a quadrilateral
 - practices vertical angles
 - uses the knowledge of angle addition
- **Identifying Attributes Of Quadrilaterals**
 - applying knowledge of quadrilaterals and their properties
- **Programming Quadrilaterals**
 - students develop relationships among the quadrilaterals, includes use of Geometer's Sketchpad

POLYGONS BIBLIOGRAPHY

Bolt, Brian. *Mathematical Activities: A resource book for teachers*. New York, NY: Press Syndicate of the University of Cambridge, 1982.

Muschla, Judith A., and Gary R. Muschla. *Geometry Teacher's Activities Kit: Ready-to-Use Lessons and Worksheets for Grades 6 – 12*. West Nyack, NY: The Center For Applied Research In Education, 2000.

CONSTRUCTING A TANGRAM

A tangram is a geometric puzzle believed to have been created in China. It consists of two large congruent right triangles, two small congruent right triangles, one medium size right triangle, a square and a parallelogram. Follow the instructions to make a tangram.

1. Use the 8-inch square provided.
2. Draw segment \overline{BD} .
3. Find the midpoint of \overline{AB} . Label it E.
4. Find the midpoint of \overline{AD} . Label it F.
5. Draw \overline{EF} .
6. Find the midpoint of \overline{EF} . Label it G.
7. Draw \overline{CG} .
8. Find the intersection of \overline{CG} and \overline{BD} . Label it H.
9. Find the midpoint of \overline{HD} . Label it J.
10. Draw \overline{FJ} .
11. Find the midpoint of \overline{HB} . Label it K.
12. Draw \overline{GK} .
13. You should now have seven pieces each called a *tan*. Cut along the lines.
14. Arrange the tans to create figures (a boat, a building, and an animal).